ECE 3043 Lecture Exam No. 2 Fall 2018
THURSDAY, OCTOBER 25, 2013, 6-6:50 PM AD LOGIN

Name

Closed Book and Note. Calculator Permitted. Four Equally Weighted Problems. All Work Must Be Shown for Credit. Box All Answers. In the questions below the term gain means the magnitude of the complex transfer function.

1. A certain filter has the complex transfer function

\[ T(s) = K \frac{1}{1 + sRC} \]

where \( K = 7.3 \), \( R = 7.3 \) k\( \Omega \), and \( C = 7.3 \) nF.

What is the order of the filter? \( \text{One, Unity} \)

What type of filter is it? \( \text{Low Pass LPF} \)

What is the minus 3 dB frequency? \( 2.992 \) Hz

What is the linear dc gain? \( 7.3 \)

What is the linear infinite frequency gain? \( 0 \)

What is the gain in dB when \( f = 4.3 \) kHz? \( 12.41 \) dB

What is the phase shift in degrees when \( f = 4.3 \) kHz? \( -55.2^\circ \)

\[ b_0 = \frac{1}{2\pi RC} = 2.992 \text{ Hz} \]

\[ T = \frac{1}{1 + \frac{jB}{b_0}} \]

\[ |T| \text{ dB} = 20 \log_{10} |T| = 12.391 \text{ dB} \]

\[ \phi = \angle T = -55.218^\circ \]
2. The component values for the filter shown below are $R = 7.3\,\text{k}\Omega$, $C = 3\,\text{nF}$, and the op amp may be considered to be ideal.

What is the order of the filter? [ ] 1 [ ] 2 [ ] 3 [ ] 4 [ ] 5 [ ] One [ ] Unity [ ] First

What type of filter is it? [ ] All Pass

What is the linear dc gain? $-1$

What is the linear infinite frequency gain? $+1$

What is the gain in dB when $f = 7.3\,\text{kHz}$? $0$ dB

What is the phase shift in degrees when $f = 7.3\,\text{kHz}$? $89.8^\circ$

Derive the complex transfer function $T(s) = \frac{V_o(s)}{V_i(s)}$. Express the answer in terms of the symbols $s$, $R$, and $C$ (not the numerical values).

via Superposition

$v_o = -\frac{R}{R} v_i + \left[1 + \frac{R}{R}\right] \frac{R}{R + \frac{1}{2C}} v_i$

\[ T = \frac{V_o}{V_i} = -1 + \frac{2\alpha\tau}{1 + 2\alpha\tau} = \frac{-1 - 2\alpha\tau + 2\alpha\tau}{1 + 2\alpha\tau} \]

\[ T = \frac{-1 + 2\alpha\tau}{1 + 2\alpha\tau} = -\frac{1 - 2\alpha\tau}{1 + 2\alpha\tau} \]
3. A certain filter has the complex transfer function

\[ T(s) = K \frac{\left( \frac{s}{\omega_0} \right)^2 - \frac{1}{Q \omega_0} + 1}{\left( \frac{s}{\omega_0} \right)^2 + \frac{1}{Q \omega_0} + 1} \]

where \( K = 73 \), \( f_0 = 7.3 \text{ kHz} \), and \( Q = 7.3 \).

What is the order of the filter? 2, 1, 1, or Dual or Second

What type of filter is it? All Pass

What is the minus 3 dB bandwidth? Doesn't Have One

What is the linear dc gain? 73

What is the linear infinite frequency gain? 73

What is the gain in dB when \( f = 11.2 \text{ kHz} \)? 37.3 dB

What is the phase shift in degrees when \( f = 11.2 \text{ kHz} \)? 17.7°
4. Determine the complex transfer function $\tilde{T}(s) = \tilde{V}_o / \tilde{V}_i$ for the circuit shown below as a ratio of two polynomials. Specify it as a function of the complex frequency, $s$, and the symbols for the resistors and capacitor. Plot the magnitude of the complex transfer function $\tilde{T}(j\omega)$ in decibels as a function of the frequency $f$ of the source as $f$ varies from $1\text{ Hz}$ to $1\text{ MHz}$. If applicable, determine the pole and zero frequencies and high and low frequency gains in decibels. Assume that the op amp is ideal. Use as the numerical values for the resistors and capacitors: $R_1 = 1\text{ k}\Omega$, $R_2 = 1\text{ k}\Omega$, $R_3 = 730\text{ k}\Omega$, $R_F = 730\text{ k}\Omega$, and $C = 4.3\text{ nF}$.

\[
\begin{align*}
T(0) &= -\frac{R_F}{R_1 + R_2} \\
T(\infty) &= -\frac{R_F}{R_1 + R_2 || R_3}
\end{align*}
\]

\[
\begin{align*}
\tilde{T}(s) &= \frac{1 + \alpha T_p}{1 + s \tau_p} = -\frac{R_F}{R_1 + R_2} \frac{1 + \alpha (R_2 + R_3) C}{1 + 2 \pi \tau C} \\
f_p &= \frac{1}{2 \pi \tau_p} = \frac{1}{2 \pi \left[ R_2 + R_1 || R_3 \right] C} = 18.5 \text{ Hz} \\
f_s &= \frac{1}{2 \pi \tau} = \frac{1}{2 \pi (R_2 + R_3) C} = 50.6 \text{ Hz} \\
|\tilde{T}(j0)|_{db} &= 20 \log_{10} \left| -\frac{R_F}{R_1 + R_3} \right| = -0.012 \text{ dB} \\
\left|\tilde{T}(j\infty)\right|_{db} &= 20 \log_{10} \left| -\frac{R_F}{R_1 + R_2 || R_3} \right| = 51.3 \text{ dB}
\end{align*}
\]
1. A certain filter has the complex transfer function

\[ T(s) = K \frac{sRC}{1 + sRC} \]

where \( K = 7.3 \), \( R = 7.3 \, \text{k}\Omega \), and \( C = 4.3 \, \text{nF} \).

What is the order of the filter? \( \text{I, 1, One, Units or First} \)

What type of filter is it? \( \text{High Pass, HPF} \)

What is the minus 3 dB frequency? \( 5.07 \, \text{kHz} \)

What is the linear dc gain? \( 0 \)

What is the linear infinite frequency gain? \( 7.3 \)

What is the gain in dB when \( f = 7.3 \, \text{kHz} \)? \( 15.6 \, \text{dB} \)

What is the phase shift in degrees when \( f = 7.3 \, \text{kHz} \)? \( 34.8^\circ \)

\[ b_0 = \frac{1}{\pi RC} = 5.07 \, \text{R} \left( 1 + \frac{1}{3} \right) \]

\[ T = \frac{1}{T} \frac{\frac{b}{b_0}}{1 + \frac{b}{b_0}} \]

\[ 20 \log_{10} |T| = 15.557 \, \text{dB} \]

\[ \angle T = 34.782^\circ \]
2. The component values for the filter shown below are $R = 7.3 \, \text{k}\Omega$, $C = 4.3 \, \text{nF}$, and the op amp may be considered to be ideal.

What is the order of the filter? \underline{1, I, One, Unity, or First}
What type of filter is it? \underline{All Pass APF}
What is the linear dc gain? \underline{+1}
What is the linear infinite frequency gain? \underline{-1}
What is the gain in dB when $f = 7.3 \, \text{kHz}$? \underline{0 \, \text{dB}}
What is the phase shift in degrees when $f = 7.3 \, \text{kHz}$? \underline{-110^\circ}

Derive the complex transfer function $T(s) = \frac{V_o(s)}{V_i(s)}$. Express the answer in terms of the symbols $s$, $R$, and $C$ (not the numerical values).

\[ T = RC \]

\[ V_0 = -\frac{R}{R} V_i + \left[1 + \frac{R}{R}\right] \frac{1}{2C} V_i \]

\[ T = \frac{\frac{V_0}{V_i}} = -1 + \frac{2}{1+\omega T} = -1 - 2T + 2 \]

\[ T = \frac{1 - 2T}{1 + 2T} \]
3. A certain filter has the complex transfer function

\[ T(s) = K \frac{-\frac{1}{Q} \frac{s}{\omega_o}}{(\frac{s}{\omega_o})^2 + \frac{1}{Q} \frac{s}{\omega_o} + 1} \]

where \( K = 7.3 \), \( f_o = 7.3 \text{kHz} \), and \( Q = 7.3 \).

What is the order of the filter? 2

What type of filter is it? Band Pass

What is the minus 3 dB bandwidth? 1

What is the linear dc gain? 0

What is the linear infinite frequency gain? 0

What is the gain in dB when \( f = 11.2 \text{kHz} \)? 0.983 dB

What is the phase shift in degrees when \( f = 11.2 \text{kHz} \)? -81.2°

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4. Determine the complex transfer function $\tilde{T}(s) = \frac{V_o}{V_i}$ for the circuit shown below as a ratio of two polynomials. Specify it as a function of the complex frequency, $s$, and the symbols for the resistors and capacitor. Plot the magnitude of the complex transfer function $\tilde{T}(j\omega)$ in decibels as a function of the frequency $f$ of the source as $f$ varies from 1 Hz to 1 MHz. If applicable, determine the pole and zero frequencies and high and low frequency gains in decibels. Assume that the op amp is ideal. Use as the numerical values for the resistors and capacitors: $R_1 = 7.3 \, k\Omega$, $R_2 = 73 \, k\Omega$, $R_F = 730 \, k\Omega$, and $C = 4.3 \, nF$.

$$\tilde{T}(0) = 1 + \frac{R_F}{R_1 + R_2}$$

$$\tilde{T}(\infty) = 1 + \frac{R_F}{R_1}$$

$$\tilde{T}(s) = \frac{1 + \frac{R_F}{R_1 + R_2}}{1 + \frac{R_1}{R_1 + R_2} R_F} \left[ 1 + \frac{R_1}{R_1 + R_2} \right] \frac{1 + \frac{R_2}{R_2 + R_F}}{1 + \frac{R_2}{R_2 + R_F}} C$$

$$f_p = \frac{1}{\pi \sqrt{C \left( R_1 + R_2 \right)}} \left( R_1 + R_2 \right) C$$

$$f_z = \frac{1}{\pi \sqrt{C \left( R_1 + R_2 \right)}} \left( R_2 R_F + R_F \right) C$$

$$|\tilde{T}(0)|_{db} = 20 \log_{10} \left| 1 + \frac{R_F}{R_1 + R_2} \right| = 20.1 \, dB$$

$$|\tilde{T}(\infty)|_{db} = 20 \log_{10} \left| 1 + \frac{R_F}{R_1} \right| = 40 \, dB$$