Feedback Amplifiers

Collection of Solved Problems

A collection of solved feedback amplifier problems can be found at the below link. The solutions are based on the use of the Mason Flow Graph described below.

http://users.ece.gate.edu/~mleach/ece3050/notes/feedback/FBExamples.pdf

Basic Description of Feedback

A feedback amplifier is one in which the output signal is sampled and fed back to the input to form an error signal that drives the amplifier. The basic block diagrams of non-inverting and inverting feedback amplifiers are shown in Fig. 1. Depending on the type of feedback, the variables \( x \), \( y \), and \( z \) are voltages or currents. The diagram in Fig. 1(a) represents a non-inverting amplifier. The summing junction at its input subtracts the feedback signal from the input signal to form the error signal \( z = x - by \) which drives the amplifier. If the amplifier has an inverting gain, the feedback signal must be added to the input signal in order for the feedback to be negative. This is illustrated in Fig. 1(b). The summing junction at the input adds the feedback signal to the input signal to form the error signal \( z = x + by \). In both diagrams, the gain around the loop is negative and equal to \( -bA \), where both \( A \) and \( b \) are positive real constants. Because the loop-gain is negative, the feedback is said to be negative. If the gain around the loop is positive, the amplifier is said to have positive feedback which causes it to be unstable.

\[
\begin{align*}
\text{(a)} & \quad x & \rightarrow & \circ & \leftarrow & \bigcirc & \rightarrow & y \\
& & & b & & A & & \\
\text{(b)} & \quad x & \rightarrow & \circ & \leftarrow & \bigcirc & \rightarrow & y \\
& & & b & & -A & & \\
\end{align*}
\]

Figure 1: Feedback amplifier block diagrams. (a) Non-inverting. (b) Inverting.

In the non-inverting amplifier of Fig. 1(a), the error signal is given by \( z = x - by \). The output signal can be written

\[
y = Az = A(x - by)
\]  \hspace{1cm} (1)

This can be solved for the gain to obtain

\[
\frac{y}{x} = \frac{A}{1 + bA}
\]  \hspace{1cm} (2)

We see that the effect of the feedback is to reduce the gain by the factor \( (1 + bA) \). This factor is called the “amount of feedback”. It is often specified in dB by the relation \( 20 \log |1 + bA| \).

In the inverting amplifier of Fig. 1(b), the error signal is given by \( z = x + by \). When \( x \) goes positive, \( y \) goes negative, so that the error signal represents a difference signal. The output signal can be written

\[
y = -Az = -A(x + by)
\]  \hspace{1cm} (3)
This can be solved for the gain to obtain

\[
\frac{y}{x} = \frac{-A}{1 + bA}
\]  

(4)

We see that the amount of feedback for the inverting amplifier is the same as for the non-inverting amplifier.

If \( A \) is large enough so that \( bA >> 1 \), the gain of the non-inverting amplifier given by Eq. (2) can be approximated by

\[
\frac{y}{x} \approx \frac{A}{b} = \frac{1}{b}
\]  

(5)

The gain of the inverting amplifier given by Eq. (4) can be approximated by

\[
\frac{y}{x} \approx -\frac{A}{bA} = -\frac{1}{b}
\]  

(6)

These are important results, for they show that the gain is set by the feedback network and not by the amplifier. In practice, this means that an amplifier without feedback can be designed without too much consideration of what its gain will be as long as the gain is high enough. When feedback is added, the gain can be reduced to any desired value by the feedback network.

The product \( bA \) in Eqs. (2) and (4) must be dimensionless. Thus if \( A \) is a voltage gain (voltage in-voltage out) or a current gain (current in-current out), then \( b \) must be dimensionless. If \( A \) is a transconductance gain (voltage in-current out), \( b \) must have the units of ohms (\( \Omega \)). If \( A \) is a transresistance gain (current in-voltage out), \( b \) must have the units siemens (\( f \)). How these are determined is illustrated below.

We have assumed so far that the gains \( A \) and \( b \) are positive real constants. In general, the gains are phasor functions of frequency. This leads to a stability problem in feedback amplifiers. As frequency is increased, \( |A| \) must eventually decrease because no amplifier can have an infinite bandwidth. The decrease in \( |A| \) is accompanied with a phase shift so that \( bA \) can be equal to a negative real number at some frequency. Suppose that \( bA = -1 \) at some frequency. Eqs. (3) and (4) show that the gain becomes infinite at that frequency. An amplifier with an infinite gain at any frequency can put out a signal at that frequency with no input signal. In this case, the amplifier is said to oscillate. It can be shown that an amplifier will oscillate if \( |bA| \geq 1 \) at any frequency for which \( bA \) is a negative real number, i.e. the phase of \( bA \) is \( \pm 180^\circ \).

In the block diagrams of Fig. 1, the input and output variables can be modeled as either a voltage or a current. It follows that there are four combinations of inputs and outputs that represent the possible types of feedback. These are summarized in Table 1 where the names for each are given. These names come from the way that the feedback network connects between the input and output stages. This is explained in the following for each type of feedback.

<table>
<thead>
<tr>
<th>Name</th>
<th>Input Variable ( x )</th>
<th>Output Variable ( y )</th>
<th>Error Variable ( z )</th>
<th>Forward Gain ( A )</th>
<th>Feedback Factor ( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series-Shunt</td>
<td>Voltage ( v )</td>
<td>Voltage ( v )</td>
<td>Voltage ( v )</td>
<td>Voltage Gain</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>Shunt-Shunt</td>
<td>Current ( i )</td>
<td>Voltage ( v )</td>
<td>Current ( i )</td>
<td>Transresistance</td>
<td>siemens (( f ))</td>
</tr>
<tr>
<td>Series-Series</td>
<td>Voltage ( v )</td>
<td>Current ( i )</td>
<td>Voltage ( v )</td>
<td>Transconductance</td>
<td>ohms (( \Omega ))</td>
</tr>
<tr>
<td>Shunt-Series</td>
<td>Current ( i )</td>
<td>Current ( i )</td>
<td>Current ( i )</td>
<td>Current Gain</td>
<td>Dimensionless</td>
</tr>
</tbody>
</table>

The Mason Signal-Flow Graph

The analysis of feedback amplifiers is facilitated by the use of the Mason signal-flow graph. A signal-flow graph is a graphical representation of a set of linear equations which can be used to
write by inspection the solution to the set of equations. For example, consider the set of equations

\[ x_2 = Ax_1 + Bx_2 + Cx_5 \]  
\[ x_3 = Dx_1 + Ex_2 \]  
\[ x_4 = Fx_3 + Gx_5 \]  
\[ x_5 = Hx_4 \]  
\[ x_6 = Ix_3 \]

where \( x_1 \) through \( x_6 \) are variables and \( A \) through \( I \) are constants. These equations can be represented graphically as shown in Fig. 2. The graph has a node for each variable with branches connecting the nodes labeled with the constants \( A \) through \( I \). The node labeled \( x_1 \) is called a source node because it has only outgoing branches. The node labeled \( x_6 \) is called a sink node because it has only incoming branches. The path from \( x_1 \) to \( x_2 \) to \( x_3 \) to \( x_6 \) is called a forward path because it originates at a source node and terminates at a non-source node and along which no node is encountered twice. The path gain for this forward path is \( AEI \). The path from \( x_2 \) to \( x_3 \) to \( x_4 \) to \( x_5 \) is called a feedback path because it originates and terminates on the same node and along which no node is encountered more than once. The loop gain for this feedback path is \( EFHC \).

Figure 2: Flow graph for the equations.

Mason’s formula can be used to calculate the transmission gain from a source node to any non-source node in a flow graph. The formula is

\[ T = \frac{1}{\Delta} \sum_k P_k \Delta_k \]  

where \( P_k \) is the gain of the \( k \)th forward path, \( \Delta \) is the graph determinant, and \( \Delta_k \) is the determinant with the \( k \)th forward path erased. The determinant is given by

\[ \Delta = 1 - \text{(sum of all loop gains)} \]
\[ + \left( \text{sum of the gain products of all possible combinations of two non-touching loops} \right) \]
\[ - \left( \text{sum of the gain products of all possible combinations of three non-touching loops} \right) \]
\[ + \left( \text{sum of the gain products of all possible combinations of four non-touching loops} \right) \]
\[ - \cdots \]  

3
For the flow graph in Fig. 2, the objective is to solve for the gain from node $x_1$ to node $x_6$. There are two forward paths from $x_1$ to $x_6$ and three loops. Two of the loops do not touch each other. Thus the product of these two loop gains appears in the expression for $\Delta$. The path gains and the determinant are given by

$$P_1 = AEI$$
$$P_2 = DI$$

$$\Delta = 1 - (B + CEFH + GH) + B \times GH$$

Path $P_1$ touches two loops while path $P_2$ touches one loop. The determinants with each path erased are given by

$$\Delta_1 = 1 - GH$$
$$\Delta_2 = 1 - (B + GH) + B \times GH$$

Thus the overall gain from $x_1$ to $x_6$ is given by

$$\frac{x_6}{x_1} = \frac{AEI \times (1 - GH) + DI \times [1 - (B + GH) + B \times GH]}{1 - (B + CEFH + GH) + B \times GH}$$

This equation can also be obtained by algebraic solution of the equations in Eqs. (7) through (11).

**Review of Background Theory**

This section summarizes several BJT small-signal ac equivalent circuits which are used to write the circuit equations in the following sections. Fig. 3(a) shows an npn BJT with Thévenin sources connected to its base and emitter and a load resistor connected to its collector. First we define the emitter intrinsic resistance $r_e$, the collector-emitter resistance $r_0$, the resistance $r'_e$, and the transconductance $G_m$. These are given by

$$r_e = \frac{V_T}{I_E}$$

$$r_0 = \frac{V_{CE} + V_A}{I_C}$$

$$r'_e = \frac{R_{th} + r_x}{1 + \beta} + r_e$$

$$G_m = \frac{a}{r'_e + R_{te}}$$

where $V_T$ is the thermal voltage, $I_E$ is the emitter bias current, $V_{CE}$ is the quiescent collector-emitter voltage, $V_A$ is the Early voltage, $I_C$ is the quiescent collector current, $r_x$ is the base spreading resistance, $\beta$ is the base-collector current gain, and $\alpha$ is the emitter-collector current gain. The latter two are related by $\alpha = \beta / (1 + \beta)$ and $\beta = \alpha / (1 - \alpha)$.

The small-signal ac Thévenin equivalent circuit seen looking into the base is shown in Fig. 3(b), where $v_b(oc)$ and $r_{ib}$ are given by

$$v_b(oc) = v_{te} \frac{r_0 + R_{tc}}{R_{te} + r_0 + R_{tc}}$$

$$r_{ib} = r_x + (1 + \beta) r_e + R_{te} \left( \frac{(1 + \beta) r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}} \right)$$

$$4$$
The small-signal ac Thévenin equivalent circuit seen looking into the emitter is shown in Fig. 3(c), where $v_{e(oc)}$ and $r_{ie}$ are given by

\begin{equation}
    v_{e(oc)} = v_{tb} \frac{r_0 + R_{tc}}{1 + \frac{r_0}{e} + \frac{r_0}{e} + \frac{R_{tc}}{1 + \beta}}
\end{equation}

\begin{equation}
    r_{ie} = \frac{r_0 + R_{tc}}{1 + \frac{r_0}{e} + \frac{r_0}{e} + \frac{R_{tc}}{1 + \beta}}
\end{equation}

The small-signal ac Norton equivalent circuit seen looking into the collector is shown in Fig. 3(d), where $i_{c(sc)}$ and $r_{ic}$ are given by

\begin{equation}
    i_{c(sc)} = G_{mb} v_{tb} - G_{me} v_{te}
\end{equation}

\begin{equation}
    r_{ic} = \frac{r_0 + r_e' || R_{te}}{1 - G_m R_{te}}
\end{equation}

The transconductances $G_{mb}$ and $G_{me}$ are given by

\begin{equation}
    G_{mb} = G_m \frac{r_0 - R_{te} / \beta}{r_0 + r_e' || R_{te}}
\end{equation}

\begin{equation}
    G_{me} = G_m \frac{r_0 + r_e' / \alpha}{r_0 + r_e' || R_{te}}
\end{equation}

The $r_0$ Approximations

In some of the examples that follow, the analysis is simplified by making use of the so-called $r_0$ approximations. That is, we assume that $r_0 \to \infty$ in all equations except in the one for $r_{ic}$. This assumption makes $G_{mb}$, $G_{me}$, $r_{ib}$, and $r_{ie}$ independent of $r_0$. In addition, it makes $G_{mb} = G_{me}$ so that we can denote $G_{mb} = G_{me} = G_m$. In this case,

\begin{equation}
    i_{c(sc)} = G_m (v_{tb} - v_{te}) = \frac{\alpha}{r_e' + R_{te}} (v_{tb} - v_{te})
\end{equation}
\[
\begin{align*}
    v_{b(oc)} &= v_{te} 	ag{33} \\
    r_{ib} &= r_x + (1 + \beta) (r_e + R_{te}) 	ag{34} \\
    v_{e(oc)} &= v_{lb} 	ag{35} \\
    r_{ie} &= r'_e 	ag{36} \\
    i_b &= \frac{i_{c(sc)}}{\beta} 	ag{37} \\
    i_e &= \frac{i_{c(sc)}}{\alpha} 	ag{38}
\end{align*}
\]

If \( v_{te} = 0 \) and \( R_{re} = 0 \), \( r_0 \) appears as a resistor from the collector to ground. If \( R_{tc} = 0 \), \( r_0 \) appears as a resistor from emitter to ground. In either case, \( r_0 \) can be easily included in the analysis by treating it as an external resistor from either the emitter or the collector to ground.

### Series-Shunt Feedback

A series-shunt feedback amplifier is a non-inverting amplifier in which the input signal \( x \) is a voltage and the output signal \( y \) is a voltage. If the input source is a current source, it must be converted into a Thévenin source for the gain to be in the form of Eq. (2). Because the input is a voltage and the output is a voltage, the gain \( A \) represents a dimensionless voltage gain. Because \( bA \) must be dimensionless, the feedback factor is also dimensionless. Two examples are given below. The first is an op-amp example. The second is a BJT example.

#### Op Amp Example

Fig. 4(a) shows an op amp with a feedback network consisting of a voltage divider connected between its output and inverting input. The input signal is connected to the non-inverting input. Because the feedback does not connect to the same terminal as the input signal, the summing is series. The feedback network connects in shunt with the output node, thus the sampling is shunt.

To analyze the circuit, we replace the circuit seen looking out of the op-amp inverting input with a Thévenin equivalent circuit with respect to \( v_o \) and the circuit seen looking into the feedback network from the \( v_o \) node with a Thévenin equivalent circuit with respect to \( i_1 \). We replace the op amp with a simple controlled source model which models the differential input resistance, the open-loop voltage gain, and the output resistance. A test source \( i_t \) is added at the output in order to calculate the output resistance. The circuit is shown in Fig. 4(b), where \( R_{id} \) is the differential input resistance, \( A_0 \) is the open-loop gain, \( R_0 \) is the output resistance of the op amp. The feedback factor \( b \) is given by

\[
b = \frac{R_1}{R_1 + R_F} 	ag{39}
\]

The error signal \( z \) in Fig. 4(b) is a voltage which we denote by \( v_e \). It is the difference between the two voltage sources in the input circuit and is given by

\[
v_e = v_b - bv_o 	ag{40}
\]

By voltage division, the voltage \( v_i \) which controls the op amp output voltage is

\[
v_i = \frac{v_e R_{id}}{R_S + R_{id} + R_1||R_F} 	ag{41}
\]
Signal tracing shows that the negative feedback has the effect of making the current $i_1$ smaller. For this reason, we will neglect the $i_1 R_1$ source in the output circuit in calculating $v_o$. By superposition, it follows that $v_o$ can be written

$$v_o = A_0 v_i \frac{(R_1 + R_F) \| R_L}{R_0 + (R_1 + R_F) \| R_L} + i_1 R_0 \| (R_1 + R_F) \| R_L$$  \hspace{1cm} (42)

To calculate the input resistance, we need the current $i_1$. It is given by

$$i_1 = \frac{v_i}{R_{id}}$$ \hspace{1cm} (43)

To simplify the equations, let us define

$$k_1 = \frac{R_{id}}{R_S + R_{id} + R_1 \| R_F}$$ \hspace{1cm} (44)

$$k_2 = \frac{(R_1 + R_F) \| R_L}{R_0 + (R_1 + R_F) \| R_L}$$ \hspace{1cm} (45)

$$R_{eq} = R_0 \| (R_1 + R_F) \| R_L$$ \hspace{1cm} (46)

The circuit equations can be rewritten

$$v_i = k_1 v_e$$ \hspace{1cm} (47)

$$v_o = k_2 A_0 v_i + i_1 R_{eq}$$ \hspace{1cm} (48)

The flow graph for these equations is shown in Fig. 5. The determinant is given by

$$\Delta = 1 - [k_1 k_2 A_0 (-b)] = 1 + bk_1 k_2 A_0 = 1 + bA$$ \hspace{1cm} (49)

From the flow graph, the voltage gain is given by

$$\frac{v_o}{v_s} = \frac{1}{\Delta} k_1 k_2 A_0 = \frac{k_1 k_2 A_0}{1 + bk_1 k_2 A_0} = \frac{A}{1 + bA}$$ \hspace{1cm} (50)

It follows that $A$ is given by

$$A = k_1 k_2 A_0$$ \hspace{1cm} (51)

This is the gain from $v_s$ to $v_o$ with $b = 0$. If $bA \gg 1$, the gain approaches

$$\frac{v_o}{v_s} \rightarrow \frac{A}{bA} = \frac{1}{b} = 1 + \frac{R_F}{R_1}$$ \hspace{1cm} (52)
This is the familiar formula for the gain of the non-inverting op amp.

From the flow graph, the output resistance is given by

\[
r_{\text{out}} = \frac{v_o}{i_t} = \frac{1}{\Delta} R_{eq} = \frac{R_0 (R_1 + R_F) R_L}{1 + bA}
\]  

(53)

Similarly, the input resistance is given by

\[
r_{\text{in}} = \left( \frac{i_1}{v_s} \right)^{-1} = \left( \frac{1}{\Delta} \frac{k_1}{R_{id}} \right)^{-1} = (1 + bA) (R_S + R_{id} + R_1 R_L)
\]  

(54)

Note that the voltage gain and the output resistance are decreased by the feedback. The input resistance is increased by the feedback. These are properties of the series-shunt topology.

**Transistor Example**

The ac signal circuit of an example BJT series-shunt feedback amplifier is shown in Fig. 6. We assume that the dc solutions to the circuit are known. The feedback network is in the form of a voltage divider and consists of resistors \( R_{F_1} \) and \( R_{F_2} \). Because the input to the feedback network connects to the \( v_o \) node, the amplifier is said to employ shunt sampling at the output. The output of the feedback network connects to the emitter of \( Q_1 \). Because this is not the node to which \( v_s \) connects, the circuit is said to have series summing at the input. The following analysis assumes the \( r_0 \) approximations for \( Q_1 \). That is, we neglect \( r_0 \) in all equations except when calculating \( r_{ic1} \).

In order for the amplifier to have negative feedback, the voltage gain from the emitter of \( Q_1 \) to the collector of \( Q_2 \) must be inverting. When the feedback signal is applied to its emitter, \( Q_1 \) is a
common-base stage which has a non-inverting voltage gain. $Q_2$ is a common-emitter stage which has an inverting gain. Thus the amplifier has an inverting voltage gain from the emitter of $Q_1$ to the collector of $Q_2$ so that the feedback is negative.

To remove the feedback, we replace the circuit seen looking out of the emitter of $Q_1$ with a Thévenin equivalent circuit with respect to $v_o$. The circuit seen looking into $R_{F1}$ from the $v_o$ node is replaced with a Thévenin equivalent circuit with respect to $i_{e1}$. Fig. 7 shows the circuit. A test current source is connected to the $v_o$ node to calculate $r_{out}$. The Thévenin voltage looking out of the emitter of $Q_1$ is given by

$$v_{te1} = v_o \frac{R_1}{R_1 + R_F} = bv_o$$

where $b$ is the feedback factor given by

$$b = \frac{R_1}{R_1 + R_F}$$

![Figure 7: Circuit with feedback removed.](image)

For the circuit of Fig. 7, the error voltage $v_e$ is given by

$$v_e = v_s - bv_o$$

Signal tracing shows that the negative feedback has the effect of making the current $i_{e1}$ smaller. For this reason, we will neglect it in calculating $v_o$. To circuit equations are

$$i_{c1(sc)} = G_{m1} (v_s - bv_o) = G_{m1}v_e$$

(58)

$$v_{th2} = -i_{c1(sc)}r_{ic1}||R_{C1}$$

(59)

$$R_{th2} = r_{ic1}||R_{C1}$$

(60)

$$i_{c2(sc)} = -G_{mb2}v_{th2}$$

(61)

$$v_o = [i_{c2(sc)} + i_t] \times r_{ic2}||R_{C2}|| (R_{F1} + R_{F2})$$

(62)

To solve for $r_{in}$, we need $i_{b1}$. If we use the $r_0$ approximations, it is given by

$$i_{b1} = \frac{i_{c1(sc)}}{\beta_1}$$

(63)
To simplify the flow graph, let us define

\[ R_{eq1} = r_{ic1} \| R_C1 \]  \hspace{1cm} (64) \\
\[ R_{eq2} = r_{ic2} \| R_C2 \| (R_{F1} + R_{F2}) \]  \hspace{1cm} (65) \\

The flow graph for the equations is shown in Fig. 8. The determinant is given by

\[ \Delta = 1 - [G_{m1} (-R_{eq1}) (-G_{m2}) R_{eq2} (-b)] = 1 + bA \]  \hspace{1cm} (66) \\

where \( A \) is given by

\[ A = G_{m1} R_{eq1} G_{m2} R_{eq2} \]  \hspace{1cm} (67) \\

This is the gain from \( v_s \) to \( v_o \) with \( b = 0 \). From the flow graph, the voltage gain is given by

\[ \frac{v_o}{v_s} = \frac{1}{\Delta} G_{m1} R_{eq1} G_{m2} R_{eq2} = \frac{A}{1 + bA} \]  \hspace{1cm} (68) \\

The output resistance is given by

\[ r_{out} = \frac{v_o}{i_t} = \frac{1}{\Delta} R_{eq2} = \frac{R_{eq2}}{1 + bA} \]  \hspace{1cm} (69) \\

The input resistance is given by \( r_{in} = v_s/i_{b1} \). Because \( v_s \) is an independent variable, it can be used to solve for \( i_{b1}/v_s \) not \( v_s/i_{b1} \). Thus the input resistance can be written

\[ r_{in} = \left( \frac{i_{b1}}{v_s} \right)^{-1} = \left( \frac{1}{\Delta} \frac{G_{m1}}{\beta_1} \right)^{-1} = \Delta \frac{\beta_1}{G_{m1}} \]  \hspace{1cm} (70) \\

But \( \beta_1/G_{m1} \) is given by

\[ \frac{\beta_1}{G_{m1}} = \beta_1 \frac{r_{e1}^' + R_{te1}}{\alpha_1} = R_S + r_x1 + (1 + \beta_1) (r_{e1} + R_{te1}) \]  \hspace{1cm} (71) \\

Thus \( r_{in} \) is given by

\[ r_{in} = (1 + bA) \left[ R_S + r_x1 + (1 + \beta_1) (r_{e1} + R_{te1}) \right] \]  \hspace{1cm} (72) \\

**Summary of the Effects of Series-Shunt Feedback**

From the examples above, it can be seen that the voltage gain is divided by the factor \( (1 + bA) \). The input resistance is multiplied by the factor \( (1 + bA) \). And the output resistance is divided by the factor \( (1 + bA) \).
Shunt-Shunt Feedback

A shunt-shunt feedback amplifier is an inverting amplifier in which the input signal $x$ is a current and the output signal $y$ is a voltage. If the input source is a voltage source, it must be converted into a Norton source for the gain to be in the form of Eq. (4). Because the input is a current and the output is a voltage, the gain $A$ represents a transresistance with the units $\Omega$. Because $bA$ must be dimensionless, the feedback factor has the units of $\Omega$. Two examples are given below. The first is an op-amp example. The second is a BJT example.

Op Amp Example

Fig. 9(a) shows an op amp with a feedback network consisting of a resistor connected between its output and its inverting input. The input signal is connected through a resistor to the inverting input. Because the feedback connects to the same terminal as the input signal, the summing is shunt. The feedback network connects in shunt with the output node, thus the sampling is shunt.

To analyze the circuit, we replace the circuit seen looking out of the $v_i$ node through $R_S$ with a Norton equivalent circuit with respect to $v_s$ and the circuit seen looking out of the $v_i$ node through $R_F$ with a Norton equivalent circuit with respect to $v_o$. This must always be done with the shunt-shunt amplifier in order for the gain with feedback to be of the form $-A/(1 + bA)$, where $A$ and $b$ are positive and the $-$ sign is necessary because the circuit has an inverting gain. In addition, we replace the circuit seen looking out of the $v_o$ node through $R_F$ with a Thévenin equivalent circuit with respect to $v_i$.

The circuit with feedback removed is shown in Fig. 9(b), where $r'_{in}$ is the input resistance seen by the source current $i_s$. The Norton current seen looking out of the $v_i$ node is represented by the $bv_o$ source, where $b$ is the feedback factor. The current $i_s$ and the feedback factor are given by

$$i_s = \frac{v_s}{R_S} \quad (73)$$

$$b = \frac{1}{R_F} \quad (74)$$

The error current $i_e$ is the total current delivered to the $v_i$ node. It is given by

$$i_e = i_s + bv_o \quad (75)$$

Because $v_o$ is negative when $i_s$ is positive, the current $bv_o$ subtracts from $i_s$ to cause $i_e$ to be decreased.

![Figure 9: (a) Shunt-shunt op amp circuit. (b) Circuit with feedback removed.](image)

Fig. 10 shows the circuit with the op amp replaced with a controlled source model which models the differential input resistance $R_{id}$, the open-loop voltage gain $A_0$, and the output resistance $R_0$. 


A test source $i_t$ is added at the output in order to calculate the output resistance. The voltage $v_i$ controls the op-amp output voltage. It is given by

$$v_i = (i_s + b v_o) R_S || R_F || R_{id} = i_e R_S || R_F || R_{id}$$

(76)

Signal tracing shows that the negative feedback has the effect of making the voltage $v_i$ smaller. For this reason, we will neglect the $v_i$ source in calculating $v_o$. It follows that $v_o$ is given by

$$v_o = -A_0 v_i R_F || R_L + i_t R_0 || R_F || R_L$$

(77)

![Figure 10: Shunt-shunt circuit with the op amp replaced with a controlled source model.](image)

To simplify the equations, let us define

$$R_{eq1} = R_S || R_F || R_{id}$$

(78)

$$R_{eq2} = R_0 || R_F || R_L$$

(79)

$$k = \frac{R_F || R_L}{R_0 + R_F || R_L}$$

(80)

The circuit equations can be rewritten

$$i_e = i_s + b v_o$$

(81)

$$v_i = i_e R_{eq1}$$

(82)

$$v_o = -k A_0 v_i + i_t R_{eq2}$$

(83)

The flow graph for these equations is shown in Fig. 11. The determinant is given by

$$\Delta = 1 - [R_{eq1} (-A_0 k) b] = 1 + b R_{eq1} A_0 k$$

(84)

From the flow graph, the transresistance gain is given by

$$\frac{v_o}{i_s} = \frac{1}{\Delta} (-R_{eq1} A_0 k) = \frac{-R_{eq1} A_0 k}{1 + b R_{eq1} A_0 k} = \frac{-A}{1 + b A}$$

(85)

It follows that $A$ is given by

$$A = R_{eq1} A_0 k$$

(86)

If $b A \gg 1$, the transresistance gain approaches

$$\frac{v_o}{i_s} \rightarrow \frac{-A}{b A} = \frac{1}{b} = -R_F$$

(87)
We note that $A$ is the negative of the gain from $i_s$ to $v_o$ calculated with $b = 0$. Also, $-bA$ is the loop gain in the flow graph.

From the flow graph, the output resistance is given by

$$r_{out} = \frac{v_o}{i_t} = \frac{1}{\Delta} R_{eq2} = \frac{R_O || R_F || R_L}{1 + bA} \tag{88}$$

Similarly, the input resistance is given by

$$r_{in}' = \frac{v_i}{i_s} = \frac{1}{\Delta} R_{eq1} = \frac{R_S || R_F || R_{id}}{1 + bA} \tag{89}$$

We note that $R_{eq2}$ is the output resistance with the $v_i$ source zeroed at the output. Also, $R_{eq1}$ is the input resistance with the $bv_o$ source zeroed at the input.

The voltage gain of the original circuit in Fig. 9(a) is given by

$$\begin{align*}
\frac{v_o}{v_s} & = \frac{i_s}{v_s} \frac{v_o}{i_s} = \frac{1}{R_S} \frac{-A}{1 + bA} \simeq \frac{1}{R_S} \frac{-A}{bA} = -\frac{R_F}{R_S} \tag{90}\end{align*}$$

where the approximation holds for $bA \gg 1$. This is the familiar gain expression for the inverting op amp amplifier. The input resistance is obtained from $r_{in}'$ with the relation

$$r_{in} = R_S + \left( \frac{1}{r_{in}'} - \frac{1}{R_S} \right)^{-1} \tag{91}$$

**Transistor Example**

Fig. 12 shows the ac signal circuit of an example BJT shunt-shunt feedback amplifier. The feedback network consists of the resistor $R_F$ which connects between the output and input nodes. Because $R_F$ connects to the output node, the amplifier is said to have shunt sampling. Because the current fed back through $R_F$ to the input node combines in parallel with the source current, the circuit is said to have shunt summing. Thus the amplifier is said to have shunt-shunt feedback. In order for the feedback to be negative, the voltage gain from $v_i$ to $v_o$ must be inverting. $Q_1$ is a common-emitter stage which has an inverting gain. $Q_2$ is a common-collector stage which has a non-inverting gain. Thus the overall voltage gain is inverting so that the feedback is negative.

Fig. 13 shows the equivalent circuit with feedback removed. The circuits seen looking out of the $v_i$ node through $R_S$ and through $R_F$, respectively, are converted into Norton equivalent circuits with respect to $v_s$ and $v_o$. The source current $i_s$ and the feedback factor $b$ are given by

$$i_s = \frac{v_s}{R_S} \tag{92}$$

$$b = \frac{1}{R_F} \tag{93}$$
Figure 12: Example BJT shunt-shunt amplifier.

Figure 13: Shunt-shunt amplifier with feedback removed.
The feedback network at the output is modeled by a Thévenin equivalent circuit with respect to $v_i$. The external current source $i_t$ is added to the circuit so that the output resistance can be calculated. Signal tracing shows that the input voltage $v_i$ is reduced by the feedback. Therefore, we neglect the effect of the $v_i$ controlled source in the output circuit when calculating $v_o$.

The circuit equations are

\[
v_{tb1} = i_e R_S || R_F = i_e R_{eq1} \tag{94}
\]

\[
i_e = i_s + b v_o \tag{95}
\]

\[
v_i = i_e R_S || r_{tib1} = i_e R_{eq2} \tag{96}
\]

\[
i_{c1(sc)} = G_{mb1} v_{tb1} \tag{97}
\]

\[
v_{tb2} = -i_{c1(sc)} r_{ic1} || R_{C1} = -i_{c1(sc)} R_{eq3} \tag{98}
\]

\[
v_{c2(oc)} = \frac{r_{t2}}{r_{t2} + r_{t02}} v_{tb2} = k_1 v_{tb2} \tag{99}
\]

\[
v_o = \frac{R_{E2} || R_F}{r_{t2} + R_{E2} || R_F} v_{c2(oc)} + i_t \times r_{t2} || R_{E2} || R_F = k_2 v_{c2(oc)} + i_t R_{eq4} \tag{100}
\]

The flow graph for the equations is shown in Fig. 14. The determinant is given by

\[
\Delta = 1 - [R_{eq1} G_{mb1} (-R_{eq3}) k_1 k_2 b] = 1 + b R_{eq1} G_{mb1} R_{eq4} k_1 k_2 \tag{101}
\]

The transresistance gain is given by

\[
\frac{v_o}{i_s} = \frac{1}{\Delta} R_{eq1} G_{mb1} (-R_{eq3}) k_1 k_2 = \frac{R_{eq1} G_{mb1} R_{eq4} k_1 k_2}{1 + b R_{eq1} G_{mb1} R_{eq3} k_1 k_2} = -\frac{A}{1 + bA} \tag{102}
\]

It follows that $A$ is given by

\[
A = R_{eq1} G_{mb1} R_{eq3} k_1 k_2 \tag{103}
\]

The input and output resistances are given by

\[
r_{in}' = \frac{v_i}{i_s} = \frac{R_{eq2}}{\Delta} = \frac{R_1 || R_F || r_{tib1}}{\Delta} \tag{104}
\]

\[
r_{out} = \frac{v_o}{i_t} = \frac{R_{eq4}}{\Delta} = \frac{r_{t2} || R_{E2} || R_F}{\Delta} \tag{105}
\]

It can be seen from these expressions that the transresistance gain, the input resistance, and the output resistance are all decreased by a factor equal to the amount of feedback.

\[
\]

Figure 14: Flow graph for the shunt-shunt amplifier.

The voltage gain of the original circuit in Fig. 12 is given by

\[
\frac{v_o}{v_s} = \frac{i_s}{v_s} \Rightarrow \frac{1}{1 - A \frac{R_F}{R_S}} \approx \frac{1}{1 - A \frac{R^2}{bA}} = -\frac{R_F}{R_S} \tag{106}
\]
where the approximation holds for $bA \gg 1$. This is the familiar gain expression for the inverting op amp amplifier. The input resistance is obtained from $r'_{in}$ with the relation

$$r_{in} = R_S + \left( \frac{1}{r'_{in}} - \frac{1}{R_S} \right)^{-1} \quad (107)$$

### Summary of the Effects of Shunt-Shunt Feedback

Notice from the examples above that the transresistance gain and the voltage gain are divided by the factor $(1 + bA)$. The input resistance $r'_{in}$ is divided by the factor $(1 + bA)$. And the output resistance is divided by the factor $(1 + bA)$.

### Series-Series Feedback

A series-series feedback amplifier is a non-inverting amplifier in which the input signal $x$ is a voltage and the output signal $y$ is a current. If the input source is a current source, it must be converted into a Thévenin source for the gain to be in the form of Eq. (2). Because the input is a voltage and the output is a current, the gain $A$ represents a transconductance with the units $\Omega$. Because $bA$ must be dimensionless, the feedback factor has the units of $\Omega$. An op-amp example is given below.

Fig. 15(a) shows an op-amp circuit in which a resistor $R_1$ is in series with the load resistor $R_L$. The voltage across $R_1$ is fed back into the inverting op-amp input. The voltage across $R_1$ is proportional to the output current $i_o$. This is said to be series sampling at the output. Because the feedback does not connect to the same op-amp input as the source, the circuit is said to have series summing. Thus the circuit is called a series-series feedback amplifier.

![Figure 15](image)

Figure 15: (a) Example series-series feedback amplifier. (b) Circuit with feedback removed.

To remove the feedback, the circuit seen looking out of the op-amp inverting input is replaced with a Thévenin equivalent circuit with respect to $i_o$. The circuit seen looking below $R_L$ is replaced with a Thévenin equivalent circuit with respect to $i_1$. The circuit with feedback removed is shown in Fig. 15(b), where the op amp is replaced with a controlled source model that models its differential input resistance $R_{id}$, its gain $A_0$, and its output resistance $R_0$. The feedback factor $b$ is given by

$$b = R_1$$

A test voltage source $v_t$ is added in series with $R_L$ to calculate the output resistance. We define the resistance $r_{out}$ as the effective series resistance in the output circuit, including $R_L$. It is given by $r_{out} = v_t / i_o$. The output resistance seen by $R_L$ is given by $r'_{out} = r_{out} - R_L$. Because $R_L$ is floating, it is not possible to label $r_{out}$ on the diagram.
Signal tracing shows that the negative feedback has the effect of reducing $i_1$. For this reason, the $i_1R_1$ source in the output circuit will be neglected in solving for $i_o$. We can write the following equations

$$v_e = v_s - bi_o$$

(108)

$$v_i = \frac{R_{id}}{R_S + R_{id} + R_1} v_e$$

(109)

$$i_o = \frac{A_0v_i}{R_0 + R_L + R_1} + \frac{v_i}{R_0 + R_L + R_1}$$

(110)

$$i_1 = \frac{v_i}{R_{id}}$$

(111)

To simplify the equations, let us define

$$k = \frac{R_{id}}{R_S + R_{id} + R_1}$$

(112)

$$R_{eq} = R_0 + R_L + R_1$$

(113)

The circuit equations can be rewritten

$$v_i = kv_e$$

(114)

$$i_o = \frac{A_0v_i}{R_{eq}} + \frac{v_i}{R_{eq}}$$

(115)

The flow graph for these equations is shown in Fig. 16. The determinant is given by

$$\Delta = 1 - \left[ k \frac{A_0}{R_{eq}} (-b) \right] = 1 + bk \frac{A_0}{R_{eq}} = 1 + bA$$

(116)

From the flow graph, the transconductance gain is given by

$$\frac{i_o}{v_s} = \frac{1}{\Delta} k \frac{A_0}{R_{eq}} = \frac{kA_0/R_{eq}}{1 + bkA_0/R_{eq}} = \frac{A}{1 + bA}$$

(117)

It follows that $A$ is given by

$$A = k \frac{A_0}{R_{eq}}$$

(118)

This is the gain from $v_s$ to $i_o$ with $b = 0$. If $bA \gg 1$, the gain approaches

$$\frac{i_o}{v_s} \to \frac{A}{bA} = \frac{1}{b} = \frac{1}{R_1}$$

(119)

This is the gain obtained by assuming the op amp is ideal. Because it has negative feedback, there is a virtual short between its two inputs so that the voltage at the upper node of $R_1$ is $v_s$ and the current through $R_1$ is $v_s/R_1$. Thus the output current is $i_o = v_s/R_1$.

The input resistance $r_{in}$ and the output resistance $r_{out}$ are given by

$$r_{in} = \left( \frac{i_1}{v_s} \right)^{-1} = \left( \frac{1}{\Delta} k \frac{R_{id}}{R_{id}} \right)^{-1} = \frac{R_{id}}{k} = \frac{1}{(1 + bA) (R_S + R_{id} + R_1)}$$

(120)

$$r_{out} = \left( \frac{i_o}{v_i} \right)^{-1} = \left( \frac{1}{\Delta} \frac{1}{R_{eq}} \right)^{-1} = \frac{1}{(1 + bA) (R_0 + R_L + R_1)}$$

(121)

Note that the effect of the feedback is to reduce the gain, increase the input resistance and increase the output resistance. In the case $bA \to \infty$, the output resistance becomes infinite and the load resistor $R_L$ is driven by an ideal current source.
Shunt-Series Feedback

A shunt-series feedback amplifier is an inverting amplifier in which the input signal \( x \) is a voltage and the output signal \( y \) is a current. If the input source is a voltage source, it must be converted into a Norton source for the gain to be in the form of Eq. (4). Because the input is a current and the output is a current, the gain \( A \) represents a dimensionless current gain. Because \( bA \) must be dimensionless, the feedback factor is dimensionless. An op-amp example is given below.

Fig. 17(a) shows an op-amp circuit in which a resistor \( R_1 \) connects from the lower node of the load resistor \( R_L \) to ground. The voltage across \( R_1 \) causes a current to flow through the feedback resistor \( R_F \) into the inverting op-amp input. The current through \( R_F \) is proportional to the output current \( i_o \). This is said to be series sampling at the output. Because the feedback connects to the same op-amp input as the source, the circuit is said to have shunt summing at the input. Thus the circuit is called a shunt-series feedback amplifier.

To analyze the circuit, we replace the circuit seen looking out of the \( v_i \) node through \( R_S \) with a Norton equivalent circuit with respect to \( v_s \) and the circuit seen looking out of the \( v_i \) node through \( R_F \) with a Norton equivalent circuit with respect to \( i_o \). This must always be done with the shunt-series amplifier in order for the gain with feedback to be of the form \(-A/(1+bA)\), where \( A \) and \( b \) are positive and the − sign is necessary because the circuit has an inverting gain. In addition, we replace the circuit seen looking through \( R_F \) into the \( v_i \) node with a Thévenin equivalent circuit with respect to \( v_i \). The circuit with feedback removed is shown in Fig. 17(b), where the op amp is replaced with a controlled source model that models its differential input resistance \( R_{id} \), its gain \( A_0 \), and its output resistance \( R_0 \). The feedback factor \( b \) is is a current divider ratio given by

\[
b = \frac{R_1}{R_1 + R_F}
\] (122)
A test voltage source $v_t$ is added in series with $R_L$ to calculate the output resistance. We define the resistance $r_{out}$ as the effective series resistance in the output circuit, including $R_L$. It is given by $r_{out} = v_t / i_o$. The output resistance seen by $R_L$ is given by $r'_{out} = r_{out} - R_L$. Because $R_L$ is floating, it is not possible to label $r_{out}$ on the diagram.

Signal tracing shows that the negative feedback has the effect of reducing $v_i$. For this reason, the $v_i$ source in the output circuit will be neglected in solving for $i_o$. The error current $i_e$ is the sum of the two current sources in the input circuit. We can write the following equations

$$i_e = i_s + bi_o$$

(123)

$$v_i = i_e R_S ((R_1 + R_F) || R_{id}) = i_e R_{eq1}$$

(124)

$$i_o = \frac{-A_0 v_i}{R_0 + R_L + R_1 || R_F} + \frac{v_t}{R_{eq2}} = \frac{-A_0 v_i}{R_{eq2}} + \frac{v_t}{R_{eq2}}$$

(125)

The flow graph for these equations is shown in Fig. 18. The determinant is given by

$$\Delta = 1 - \left[ R_{eq1} \left( \frac{-A_0}{R_{eq2}} \right) b \right] = 1 + b R_{eq1} \frac{A_0}{R_{eq2}} = 1 + bA$$

(126)

From the flow graph, the current gain is given by

$$\frac{i_o}{i_s} = \frac{1}{\Delta} R_{eq1} \left( \frac{-A_0}{R_{eq2}} \right) = \frac{-R_{eq1} A_0 / R_{eq2}}{1 + bR_{eq1} A_0 / R_{eq2}} = \frac{-A}{1 + bA}$$

(127)

It follows that $A$ is given by

$$A = R_{eq1} \frac{A_0}{R_{eq2}}$$

(128)

This is the negative of the gain from $i_s$ to $i_o$ with $b = 0$. If $bA \gg 1$, the gain approaches

$$\frac{i_o}{i_s} \sim \frac{-A}{bA} = -\frac{1}{b} = - \left( 1 + \frac{R_F}{R_1} \right)$$

(129)

This is the gain obtained by assuming the op amp is ideal. In this case, there is a virtual ground at its inverting input which causes $i_e = i_s + bi_o = 0$. Solution for $i_o$ yields $i_o = -i_s/b$.

![Flow graph for the shunt-series amplifier.](image)

The input resistance $r'_{in}$ and the output resistance $r_{out}$ are given by

$$r'_{in} = \frac{v_i}{i_s} = \frac{1}{\Delta} R_{eq1} = \frac{R_S ((R_1 + R_F) || R_{id})}{1 + bA}$$

(130)

$$r_{out} = \left( \frac{i_o}{v_t} \right)^{-1} = \left( \frac{1}{\Delta} \right)^{-1} = \frac{1}{(1 + bA) (R_0 + R_L + R_1 || R_F)}$$

(131)
Note that the effect of the feedback is to reduce the gain, decrease the input resistance and increase the output resistance. In the case $bA \to \infty$, the output resistance becomes infinite and the load resistor $R_L$ is driven by an ideal current source. The gain from $v_s$ to $i_o$ in the original circuit, the input resistance $r_{in}$, and the output resistance $r'_{out}$ seen by $R_L$ are given by

$$\frac{i_o}{v_s} = \frac{i_s}{v_s} \frac{i_o}{i_s} = \frac{1}{R_S} \frac{-A}{1 + bA} = \frac{1}{R_S} \frac{-A}{1 + bA}$$

(132)

$$r_{in} = R_S + \left( \frac{1}{r'_{in}} - \frac{1}{R_S} \right)^{-1}$$

(133)

$$r'_{out} = r_{out} - R_L$$

(134)
Stop Here

Figure 10 shows the circuit diagram of a series-series feedback amplifier, where the bias circuits have been omitted for simplicity. The feedback network consists of resistors \( R_{F1} \) and \( R_{F2} \). This network samples the voltage at the collector of \( Q_2 \) and feeds a voltage back into the emitter of \( Q_1 \). The output voltage from the circuit is the voltage at the emitter of \( Q_1 \). Thus the feedback network does not sample an output voltage. Instead, it samples a voltage which is proportional to the output current, i.e. it samples a voltage proportional to \( i_{c2} = \alpha_2 i_o \). Because \( i_o \) is the current through the load resistor \( R_{E2} \), it follows that the feedback is proportional to the load current. This is called series sampling. The input summing is identical to that for the circuit of Fig. 4 which is series summing. Therefore, the circuit is a series-series feedback amplifier.

XXXExp17f10.wmf f10Series-series feedback amplifier.

To analyze the circuit, first remove the feedback. The circuit with feedback removed is shown in Fig. 11. An external voltage source \( v_i \) is included in series with \( R_{E2} \) in order to calculate the output resistance \( r_{out} \). Notice that \( r_{out} \) is labeled looking into \( R_{E2} \) from signal ground. To simplify the analysis, it will be assumed that \( r_0 = \infty \) for both BJTs. The Thévenin equivalent circuit seen looking into the emitter of \( Q_2 \) consists of the voltage source \( v_{ib2} = -i_{c1} R_{C1} \) in series with the resistor \( r_{ie2} \). To solve for \( i_o/v_i, r_{in} \), and \( r_{out} \), we can write the following equations:

\[
i_o = i_{c2} = \frac{v_t - v_{ib2}}{r_{ie2} + R_{E2}} = \frac{v_t + i_{c1} R_{C1}}{r_{ie2} + R_{E2}}
\]

(135)

\[
i_{c1} = g_m (v_{ib1} - v_{ie1}) = g_m \left( v_i - i_{c2} \frac{R_{C2}}{R_{C2} + R_{F1} + R_{F2}} R_{F2} \right)
\]

(136)

\[
i_{c2} = \alpha_2 i_o \quad i_{b1} = \frac{i_{c1}}{\beta_1}
\]

(137)

where \( R_{ib1} = R_1, R_{ie1} = R_{F2} \parallel (R_{F1} + R_{C2}) \), and

\[
r_{ie2} = r_e + \frac{r_x + R_{ib2}}{1 + \beta_2} = \frac{r_x + R_{C1}}{1 + \beta_2}
\]

(138)

XXXExp17f11.wmf f11Series-series amplifier with feedback removed.

The flow graph for the equations is given in Fig. 12. The determinant is given by

\[
\Delta = 1 - \frac{R_{C1}}{r_{ie2} + R_{E2}} \times \alpha_2 \times \frac{-g_m R_{C2} R_{F2}}{R_{C2} + R_{F1} + R_{F2}}
\]

(139)

The transconductance gain is given by

\[
\frac{i_o}{v_i} = \frac{1}{\Delta} \times g_m \times \frac{R_{C1}}{r_{ie2} + R_{E2}}
\]

(140)

The output resistance is given by

\[
r_{out} = \left( \frac{i_o}{v_i} \right)^{-1} = \Delta \times (r_{ie2} + R_{E2})
\]

(141)

The input resistance is given by

\[
r_{in} = \left( \frac{i_{b1}}{v_i} \right)^{-1} = \Delta \times \frac{\beta_1}{g_m} = \Delta \times (R_1 + r_{ib1})
\]

(142)

where the alternate relation \( g_m = \beta_1/(R_1 + r_{ib1}) \) has been used in the expression for \( r_{in} \) and \( r_{ib1} = r_x + (1 + \beta_1)(r_e + R_{ib1}) \). It is seen from these relations that the transconductance gain is
reduced by the amount of feedback, whereas both the output resistance and the input resistance are increased by the amount of feedback. These are all properties of series-series feedback.

The feedback network consists of resistor $R_F$. The feedback network samples a voltage which is proportional to $i_e$, i.e. it samples a voltage proportional to $i_{e2} = i_o/\alpha_2$. Because $i_o$ is the current through the load resistor $R_C$, it follows that the feedback is proportional to the load current. This is called series sampling. The input summing is identical to that for the circuit of Fig. 7 which is shunt summing. Therefore, the circuit is a shunt-series feedback amplifier.

To solve for the load current and input resistance, it will be assumed that $r_0 = \infty$ for both transistors. This simplifying assumption forces the output resistance to be infinite. A separate solution will be given to solve for the output resistance. To analyze the circuit, first remove the feedback. The circuit with feedback removed is shown in Fig. 14. The Thévenin voltage seen looking out of the base of $Q_2$ is $v_{b2} = -i_{c1}R_C$. To solve for $i_o/i_i$ and $r_{in}$, we can write

$$i_o = G_{m2}(v_{b2} - v_{e2}) = G_{m2}(-i_{c1}R_C - v_{b1}\frac{R_E}{R_F + R_E})$$

$$i_{c1} = G_{m1}v_{b1}$$

$$v_{b1} = i_e|R_1||R_F + R_E||r_{ib1}$$

$$i_e = i_i + i_{e2}\frac{R_E}{R_F + R_E}$$

where $i_e$ is the error current, $R_{ib1} = 0$, $R_{te1} = R_{E1}$, $R_{ib2} = R_{C1}$, $R_{te2} = R_{E2}||R_F$, and

$$r_{ib1} = r_{x1} + (1 + \beta_1)(r_{e1} + R_{E1})$$

The flow graph for the equations is given in Fig. 15. The determinant of the graph is given by

$$\Delta = 1 - \left[G_{m1}(-G_{m2}R_C) - \frac{G_{m2}R_E}{R_F + R_E} \right] \frac{1}{\alpha_2} \times \frac{R_E}{R_F + R_E} [R_1||R_F + R_E]_2$$
The current gain is given by

\[
\frac{i_o}{i_i} = \frac{1}{\Delta} [R_1 \parallel (R_F + R_{E2})] r_{ib1} \left[ G_{m1} (-G_{m2} R_C1) - \frac{G_{m2} R_{E2}}{R_F + R_{E2}} \right] \tag{152}
\]

The input resistance is given by

\[
r_{in} = \frac{v_{b1}}{i_i} = \frac{1}{\Delta} [R_1 \parallel (R_F + R_{E2})] r_{ib1} \tag{153}
\]

Thus both the current gain and the input resistance are decreased by the amount of feedback. These are properties of the shunt-series topology.

It follows from Eqs. (103) and (??) that \( A \) and \( b \) are given by

\[
A = [R_1 \parallel (R_F + R_{E2})] \left[ G_{m1} (-G_{m2} R_C1) - \frac{G_{m2} R_{E2}}{R_F + R_{E2}} \right] \tag{154}
\]

\[
b = -\frac{1}{\alpha_2} \times \frac{R_{E2}}{R_F + R_{E2}} \tag{155}
\]

Note that \( b \) is negative. Because \( A \) is also negative, the product \( bA \) is positive. If this product is not positive, the amplifier would have positive feedback rather than negative feedback. Eq. (5) can be used to write the approximate current gain as

\[
\frac{i_o}{i_i} \approx \frac{1}{b} = -\alpha_2 \left( 1 + \frac{R_F}{R_{E2}} \right) \tag{156}
\]

XXXExp17f16.wmf f16Circuit for calculating \( r_{out} \).

Because it is assumed that \( r_{02} = \infty \) in the above analysis, it follows that \( r_{out} = \infty \). To obtain a finite \( r_{out} \), \( r_{02} \) must be added from the collector to the emitter of \( Q_2 \) as shown in the circuit with feedback removed in Fig. 16. Note the use of the primes to denote the collector and emitter currents of \( Q_2 \) that are inside the resistor \( r_{02} \). In the circuit, the source \( i_t \) is set to zero and a current source \( i_t \) is added in series with the collector of \( Q_2 \). Because \( i_t = 0 \), the circuit seen looking into \( R_F \) from the emitter of \( Q_2 \) can be represented by the resistance \( R_F + R_1 \| r_{ib1} \) to ground. The output resistance calculated from this circuit is the output resistance seen by the load resistor \( R_{C2} \) in Fig. (14). The analysis can be simplified by replacing the circuit seen looking out of the base of \( Q_2 \) by a Thévenin equivalent circuit. This is shown in Fig. 17.

XXXExp17f17.wmf f17Circuit for calculating \( r_{out} \) after making a Thévenin equivalent circuit looking out of the base of \( Q_2 \).

The transresistance \( R_{m1} \) is given by

\[
R_{m1} = \frac{v_{b2}}{i_{e2}} = \frac{v_{b2}}{i_{c1}} \times \frac{i_{c1}}{v_{b1}} \times \frac{v_{b1}}{i_{e2}} = (-R_{C1}) \times G_{m1} \times \left[ \frac{R_{E2}}{R_F + R_{E2}} \times R_1 \| (R_F + R_{E2}) \| r_{ib1} \right] \tag{157}
\]

where \( R_{ib1} = 0 \) and \( R_{te1} = R_{E1} \). For this circuit

\[
v_{e2} = i_{02} r_{02} + i_{e2} R_{te2} \tag{158}
\]

\[
\dot{i}_{02} = i_t - \dot{i}_{c2} \tag{159}
\]

\[
\dot{i}_{e2} = G_{m2} (-R_{m1} \dot{i}_{c2} - i_{02} R_{te2}) \tag{160}
\]
\[ i_{c2} = i_{c2} + i_{b2} = i_{c2} + \frac{i_{c2}}{\beta_2} \]  

(161)

where \( R_{tc2} = [R_F + R_1 \parallel r_{bb1}] \parallel R_{E2} \) and \( i_{c2} = i_t \).

XXXExp17f18.wmf f18Flow graph for calculating \( r_{out} \).

The flow-graph for these equations is shown in Fig. 18. There are two loops. The determinant is given by

\[ \Delta = 1 - \left[ G_{m2}R_{tc2} - G_{m2}R_m \frac{1}{\beta_2} \right] \]  

(162)

There are four forward paths from \( i_t \) to \( v_{c2} \), two of which touch both loops, one which touches only one loop, and one which touches only the other loop. The output resistance is given by

\[ r_{out} = \frac{v_c}{i_t} = \frac{1}{\Delta} \left[ r_{02} \Delta_1 + r_{02}G_{m2}R_m + R_{tc2}\Delta_2 - G_{m2}R_{tc2}^2 \frac{1}{\beta_2} \right] \]  

(163)

where \( \Delta_1 = 1 + G_{m2}R_m/\beta_2 \) and \( \Delta_2 = 1 - G_{m2}R_{tc2} \). It is straightforward to show that (122) reduces to

\[ r_{out} = \frac{r_{02} \left( 1 + G_{m2}R_m/\alpha_2 \right) + (r_{ic2} \parallel R_{tc2})}{1 - G_{m2} \left( R_{tc2} - R_m/\beta_2 \right)} \]  

(164)

where \( r_{ic2} = r_{c2} + (R_{c1} + r_x) / (1 + \beta_2) \).

### Design Examples

#### Series-Shunt

Figure 19 shows the circuit diagram of a practical series-shunt amplifier. The circuit is to be designed for the following specifications: voltage gain \( v_o/v_i = 10 \), \( R_L = R_{in} \approx 10 \text{ k}\Omega \), \( I_{C1} = 0.5 \text{ mA} \), \( I_{C2} = 3 \text{ mA} \), \( V^+ = -V^- = 15 \text{ V} \). It may be assumed that \( V_{BE1} = V_{EB2} = 0.65 \text{ V} \) and \( \beta_1 = \beta_2 = \infty \) for the calculations.

![Series-shunt amplifier circuit diagram](image)

**Figure 19: Series-shunt amplifier.**

Let \( R_8 = 100 \text{ \Omega} \). The voltage across \( R_7 \) is \( I_{E2}R_8 + V_{EB2} = 0.95 \text{ V} \). Thus \( R_7 = 0.95/0.5 = 1.9 \text{ k}\Omega \). Let \( V_{E1} = -5 \text{ V} \), \( V_{C2} = 0 \text{ V} \), and \( R_5 = 20 \text{ k}\Omega \). The current through \( R_5 \) is 5/20 = 0.25 mA. The current through \( R_4 \) is \( I_{E1} + 0.25 = 0.75 \text{ mA} \). The voltage across \( R_4 \) is 10 V. Thus \( R_4 = 10/0.75 = 13.3 \text{ k}\Omega \). The current through \( R_6 \) is 3 – 0.25 = 2.75 mA. The voltage across \( R_6 \) is
Thus, the closed-loop gain is given by \((5 + 15)\). If the open-loop gain of the amplifier is very large, the closed-loop gain is given by \(1 + R_5/(R_3|R_4)\). Let us use this gain expression to design for a gain of \(10\sqrt{2}\), i.e. 3 dB higher than the specified value, in hopes that the actual gain will be close to 10. It follows that \(1 + R_5/(R_3|R_4) = 10\sqrt{2}\). Solution for \(R_3\) yields \(R_3 = 1.72\) kΩ. If the circuit oscillates, a small capacitor, typically 10 pF can be connected from collector to base of \(Q_2\). For acceptable low frequency response above the frequency \(f_c\), the capacitors should satisfy \(C_1 > 1/(2\pi f_c R_{in})\), \(C_2 > 1/(2\pi f_c R_L)\), and \(C_3 > 1/[2\pi (R_3 + R_4||R_5||r_{e1})]\), where \(r_{e1} = V_T/I_E1\). For \(f_c = 10\) Hz, acceptable values are \(C_1 = C_2 = 10\) μF and \(C_3 = 22\) μF.

Once the circuit is assembled, \(R_3\) can be tweaked to fine tune the gain. Either \(R_1\), \(R_2\), or both can be tweaked to adjust the dc bias voltage at the collector of \(Q_2\).

**Shunt-Shunt**

Figure 20 shows the circuit diagram of a practical shunt-shunt amplifier. The circuit is to be designed for the following specifications: voltage gain \(v_0/v_1 = -10\), \(R_{in} = 10\) kΩ, \(R_L = 100\) kΩ, \(I_{C1} = 0.5\) mA, \(I_{C2} = 4\) mA, \(V^+ = -V^- = 15\) V. It may be assumed that \(V_{BE1} = V_{BE2} = 0.65\) V and \(\beta_1 = \beta_2 = \infty\) for the calculations. Because \(v_1 = i_1 R_{in}\), it follows that the transresistance gain is \(v_0/i_1 = -100\) kΩ.

Let us choose the dc voltage at the emitter of \(Q_1\) to be \(-10\) V and the dc voltage at the emitter of \(Q_2\) to be \(+5\) V. The current through \(R_5\) and \(R_8\) is 0.5 mA, thus \(R_5 = (-10 + 15)/0.5\) m = 10 kΩ and \(R_8 = (15 - 5.65)/0.5\) m = 18 kΩ. The current through \(R_7\) is 4 mA, thus \(R_7 = (5 + 15)/4\) m = 5 kΩ. Let us pick the current through \(R_2\) and \(R_3\) to be 0.05 mA. If \(\beta_{1\text{min}} = 100\), this current is 10 times \(I_{B1\text{max}}\). Thus \(R_2 = (15 + 9.35)/0.05\) m = 487 kΩ. Let us choose the 5% value \(R_2 = 470\) kΩ. It follows that the current through \(R_2\) is \((15 + 9.35)/470\) m = 0.0518 mA. Thus \(R_3 = (-9.35 + 15)/0.0518\) m = 109 kΩ. Let us choose the 5% value \(R_3 = 110\) kΩ. The smaller the value of \(R_4\), the higher the open-loop gain. To prevent possible oscillation problems, the value of \(R_3\) should not be too small. Let us pick \(R_3 = 100\) Ω. If the open-loop gain is very high, the input resistance is \(R_{in} \simeq R_1\). Thus let us choose \(R_1 = 10\) kΩ to meet the input resistance specification. If the open-loop gain is infinite, the closed-loop voltage gain is \(v_0/v_1 = -R_6/R_1\). For a gain of \(-10\), we must have \(R_6 = 100\) kΩ. To correct for a non-infinite open-loop gain, \(R_6\) should

![Figure 20: Shunt-shunt amplifier.](image-url)
be larger than this. Let us choose the value \( R_6 = 120 \, \text{k}\Omega \), which is 20% larger. For the capacitors, let \( C_1 = C_2 = C_4 = 1 \mu\text{F} \) and \( C_3 = 100 \mu\text{F} \).

**Series-Series**

Figure 21 shows the circuit diagram of a series-series amplifier. The circuit is to be designed for the following specifications: transconductance gain \( \frac{i_{c2}}{i_1} = 2 \, \text{mA/V} \), \( R_{in} \approx 10 \, \text{k}\Omega \), \( R_L = 10 \, \text{k}\Omega \), \( I_{C1} = 0.5 \, \text{mA} \), \( I_{C2} = 4 \, \text{mA} \), \( V^+ = -V^- = 15 \, \text{V} \). It may be assumed that \( V_{BE1} = V_{BE2} = 0.65 \, \text{V} \) and \( \beta_1 = \beta_2 = \infty \) for the calculations.

![Series-shunt amplifier](image)

Figure 21: Series-shunt amplifier.

Let us choose the following dc voltages: \( V_{E1} = V_{C2} = -10 \, \text{V} \) and \( V_{C1} = 2 \, \text{V} \). The current through \( R_6 \) and \( R_9 \) is 4 mA, thus \( R_6 = (-10 + 15) / 4m = 1.3 \, \text{k}\Omega \) and \( R_9 = (15 - 2.65) / 4m = 3 \, \text{k}\Omega \). The current through \( R_4 \) and \( R_8 \) is 0.5 mA, thus \( R_4 = (-10 + 15) / 0.5m = 10 \, \text{k}\Omega \) and \( R_8 = (15 - 2) / 0.5m = 26 \, \text{k}\Omega \). Because \( V_{B1} = -10 + 0.65 = -9.35 \, \text{V} \), we have \( 30R_2 / (R_1 + R_2) - 15 = -9.35 \). Also, \( R_1R_2 / (R_1 + R_2) = 10 \, \text{k}\Omega \). These equations can be solved for \( R_1 \) to obtain 53 k\Omega. This is not a standard 5% value. Let us pick \( R_1 = 56 \, \text{k}\Omega \). For \( V_{B1} = -9.35 \, \text{V} \), it follows that \( R_2 = 13 \, \text{k}\Omega \). Let us pick \( R_5 = 1.3 \, \text{k}\Omega \). If the open-loop gain is infinite, the transconductance gain would be given by \( \frac{i_{c2}}{V_1} = \frac{(R_6 + R_5 + R_3)\|R_4)}{(R_6 \times R_3)\|R_4} = 0.002 \). For the values for \( R_5 \) and \( R_6 \), it follows from this equation that \( R_3\|R_4 = 1625 \, \text{\Omega} \), thus \( R_3 = 1.94 \, \text{k}\Omega \). To account for a non-infinite open-loop gain, the value \( R_3 = 1.3 \, \text{k}\Omega \) which is about 33% smaller than the 1.94 k\Omega value.

**Shunt-Series**

Figure 22 shows the circuit diagram of a shunt-series amplifier. The circuit is to be designed for the following specifications: current gain \( \frac{i_{c2}}{i_1} = -10 \), \( R_{in} \approx 10 \, \text{k}\Omega \), \( R_L = 10 \, \text{k}\Omega \), \( I_{C1} = 0.5 \, \text{mA} \), \( I_{C2} = 4 \, \text{mA} \), \( V^+ = -V^- = 15 \, \text{V} \), \( V_{C2} = +5 \, \text{V} \), \( V_{C1} = -5 \, \text{V} \), and \( V_{B1} = V_{BE2} = -10 \, \text{V} \). It may be assumed that \( V_{BE1} = V_{BE2} = 0.65 \, \text{V} \) and \( \beta_1 = \beta_2 = \infty \) for the calculations.

Both \( R_7 \) and \( R_9 \) conduct a current of 4 mA, thus \( R_7 = (-5 - 0.65 + 15) / 4m = 2.34 \, \text{k}\Omega \) and \( R_9 = (15 - 5) / 4m = 2.5 \, \text{k}\Omega \). Both \( R_5 \) and \( R_8 \) conduct a current of 0.5 mA, thus \( R_5 = (-10 - 0.65 + 15) / 0.5m = 8.7 \, \text{k}\Omega \) and \( R_8 = (15 + 5) / 0.5m = 40 \, \text{k}\Omega \). Let us take the current through \( R_2 \) to be 0.05 mA, i.e. about 10 times the maximum expected value of \( I_{B1} \) for \( \beta_{\text{min}} = 100 \). Thus we have \( R_2 = (15 + 10) / 0.05m = 500 \, \text{k}\Omega \). Also, \( R_3 = (-10 + 15) / 0.05m = 100 \, \text{k}\Omega \). If
we assume that the open-loop gain is infinite, the input resistance is $R_1$ and the current gain is $i_{o2}/i_1 = -(1 + R_6/R_7)$. Thus, we pick $R_1 = 10$ kΩ and $(1 + R_6/1.25k) = 10$. Solution for $R_6$ yields $R_6 = 21.1$ kΩ. We will pick the standard value $R_6 = 22$ kΩ. To complete the design, we pick $C_1 = C_2 = 10 \mu$F, $C_3 = 100 \mu$F, $C_4 = 10 \mu$F, and $R_4 = 100$ Ω.
Series Shunt Amplifier

\[ R_p(A, B) := \frac{A \cdot B}{A + B} \]

\[ V_T := 25.9 \text{mV} \]

\[ V_{BE} := 0.65 \text{V} \]

\[ V_{EB} := 0.65 \text{V} \]

\[ R_L := 10 \text{k}\Omega \]

\[ V_{\text{plus}} := 15 \text{V} \quad V_{\text{minus}} := -15 \text{V} \quad R_8 := 0 \quad R_1 := 28 \text{k}\Omega \quad R_2 := 15 \text{k}\Omega \]

\[ R_3 := 1.8 \text{k}\Omega \quad R_4 := 13 \text{k}\Omega \quad R_5 := 20 \text{k}\Omega \quad R_6 := 5.1 \text{k}\Omega \quad R_7 := 1.8 \text{k}\Omega \quad R_8 := 100 \Omega \]

\[ V_{B1} := V_{\text{plus}} \cdot \frac{R_2 - R_1}{R_1 + R_2} = -4.535 \text{V} \quad V_{E1} := V_{B1} - V_{BE} = -5.185 \text{V} \]

\[ \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \left( \frac{R_6}{R_4 + R_5 + R_6} \cdot \frac{R_6}{R_4 + R_5 + R_4} \right)^{-1} \begin{pmatrix} V_{E1} - V_{\text{minus}} \\ V_{EB} \end{pmatrix} = \begin{pmatrix} 0.53 \\ 3.034 \end{pmatrix} \text{mA} \]

\[ V_{E1} := I_1 \cdot R_p(R_4, R_5 + R_6) + I_2 \cdot \left( \frac{R_6}{R_4 + R_5 + R_6} \cdot R_4 \right) + V_{\text{minus}} = -5.185 \text{V} \]

\[ V_{C1} := V_{\text{plus}} - I_1 \cdot R_7 = 14.047 \text{V} \quad V_{B2} := V_{C1} = 14.047 \text{V} \quad V_{E2} := V_{B2} + V_{EB} = 14.697 \text{V} \]

\[ V_{C2} := I_2 \cdot R_p(R_6, R_4 + R_5) + I_1 \cdot \frac{R_4}{R_4 + R_5 + R_6} \cdot R_6 + V_{\text{minus}} = -0.678 \text{V} \]

\[ r_{e1} := \frac{V_T}{I_1} \quad G_{m1} := \frac{1}{r_{e1} + R_p(R_5, R_p(R_3, R_4))} \quad r_{e2} := \frac{V_T}{I_2} \quad G_{m2} := \frac{1}{r_{e2} + R_8} \]

\[ A := G_{m1} \left( G_{m2} \cdot R_7 \cdot R_p(R_6, R_L) \cdot R_5 + R_p(R_3, R_4) \right) + \frac{R_p(R_3, R_4) \cdot R_p(R_6, R_L)}{R_p(R_3, R_4) + R_5 + R_p(R_6, R_L)} = 32.128 \]
\[ b := \frac{R_p(R_3, R_4)}{R_5 + R_p(R_3, R_4)} = 0.073 \quad \frac{1}{b} = 13.65 \quad A_V := \frac{A}{1 + b \cdot A} = 9.58 \]

Put cap to ground from emitter of Q2

\[ G_{m2} := \frac{1}{r_{e2}} \]

\[ A := G_{m1} \left( G_{m2} R_7 R_p(R_p(R_6, R_L), R_5 + R_p(R_3, R_4)) + \frac{R_p(R_3, R_4) R_p(R_6, R_L)}{R_p(R_3, R_4) + R_5 + R_p(R_6, R_L)} \right) = 406.789 \]

\[ b := \frac{R_p(R_3, R_4)}{R_5 + R_p(R_3, R_4)} = 0.073 \quad A_V := \frac{A}{1 + b \cdot A} = 13.206 \]
Series Shunt Feedback Amplifier
Shunt-Shunt Amplifier

\[ V_{\text{plus}} := 15 \text{V} \quad V_{\text{minus}} := -15 \text{V} \quad \beta := 10^{200} \quad R_G := 0 \Omega \quad R_1 := 10k\Omega \]

\[ R_2 := 487k\Omega \quad R_3 := 109k\Omega \quad R_4 := 100\Omega \quad R_5 := 10k\Omega \quad R_6 := 120k\Omega \]

\[ R_7 := 5k\Omega \quad R_8 := 18k\Omega \quad R_L := 10k\Omega \quad V_{\text{BE}} := 0.65\text{V} \quad V_T := 25.9\text{mV} \]

\[ V_{B1} := \frac{R_3 - R_2}{R_2 + R_3} \cdot V_{\text{plus}} = -9.513\text{V} \quad V_{E1} := V_{B1} - V_{\text{BE}} = -10.163\text{V} \]

\[ I_{E1} := \frac{V_{E1} - V_{\text{minus}}}{R_5} = 0.484\text{mA} \quad I_{C1} := I_{E1} \quad V_{C1} := V_{\text{plus}} - I_{C1} \cdot R_8 = 6.294\text{V} \]

\[ V_{B2} := V_{C1} \quad V_{E2} := V_{B2} - V_{\text{BE}} = 5.644\text{V} \quad I_{E2} := \frac{V_{E2} - V_{\text{minus}}}{R_7} = 4.129\text{mA} \]

\[ r_{e1} := \frac{V_T}{I_{E1}} = 53.55\Omega \quad r_{e2} := \frac{V_T}{I_{E2}} = 6.273\Omega \quad R_p(A, B) := \frac{A - B}{A + B} \]

\[ G_{m1} := \frac{1}{r_{e1} + R_p(R_4, R_5)} \quad G_{m2} := \frac{1}{r_{e2} + R_p(R_7, R_p(R_L, R_6))} \]

\[ R_{23} := R_p(R_2, R_3) \quad R_1 := R_p(R_1, R_{23}) \quad R_{E2} := R_p(R_7, R_L) = 3.333\text{k\Omega} \]

\[ A := R_p(R_1, R_6) \left[ G_{m1} \left( -R_8 \right) \frac{R_p(R_{E2}, R_6)}{r_{e2} + R_p(R_{E2}, R_6)} + \frac{R_p(r_{e2} \cdot R_{E2})}{R_6 + R_p(r_{e2} \cdot R_{E2})} \right] = -9.849 \times 10^5 \text{\Omega} \]
\[ b := \frac{-1}{R_6} \quad \frac{1}{b} = -120 \cdot k\Omega \]

\[ A_r := \frac{A}{1 + A \cdot b} = -106.967 \cdot k\Omega \]
Shunt Shunt Feedback Amplifier
Series Series Feedback

\[ V_{\text{plus}} := 15\text{V} \quad V_{\text{minus}} := -15\text{V} \quad V_{\text{BE}} := 0.65\text{V} \quad V_{\text{EB}} := 0.65\text{V} \quad V_T := 25.9\text{mV} \]

\[ \beta := 10^{307} \quad R_p(A,B) := \frac{A \cdot B}{A + B} \quad R_1 := 53k\Omega \quad R_2 := 13k\Omega \quad R_3 := 1.94k\Omega \]

\[ R_4 := 10k\Omega \quad R_5 := 1.3k\Omega \quad R_6 := 1.25k\Omega \quad R_7 := 26k\Omega \quad R_8 := 3.0875k\Omega \]

\[ R_L := 10k\Omega \quad V_{B1} := V_{\text{plus}} \cdot \frac{R_2 - R_1}{R_2 + R_1} = -9.091\text{V} \quad V_{E1} := V_{B1} - V_{\text{BE}} = -9.741\text{V} \]

\[
\begin{pmatrix}
I_1 \\
I_2
\end{pmatrix}
= \begin{bmatrix}
\frac{R_4(R_5 + R_6)}{R_4 + R_5 + R_6} & \frac{R_4 \cdot R_6}{R_4 + R_5 + R_6} \\
-\frac{R_7}{R_8}
\end{bmatrix}^{-1} \begin{pmatrix}
V_{E1} - V_{\text{minus}} \\
V_{\text{EB}}
\end{pmatrix} = \begin{pmatrix}
0.525 \\
4.209
\end{pmatrix} \text{mA}
\]

\[ r_{e1} := \frac{V_T}{I_1} \quad r_{e2} := \frac{V_T}{I_2} \quad R_{F2} := R_p(R_3,R_4) \quad R_{F1} := R_5 \quad R_{C2} := R_6 \]

\[ R_{te1} := R_p(R_{F2},R_{F1} + R_{C2}) \quad R_{te2} := R_p(R_8,R_L) \quad R_{C1} := R_7 \]
\[ G_{m1} := \frac{1}{r_{e1} + R_{te1}} \quad G_{m2} := \frac{1}{r_{e2} + R_{te2}} \]

\[ A := G_{m1} \cdot \frac{R_{C1}}{r_{e2} + R_{te2}} = 0.011 \frac{1}{\Omega} \quad b := \frac{R_{C2} \cdot R_{F2}}{R_{C2} + R_{F1} + R_{F2}} = 486.489 \Omega \]

\[ \frac{1}{b} = 2.056 \times 10^{-3} \frac{1}{\Omega} \quad A_m := \frac{A}{1 + b \cdot A} = 1.72 \times 10^{-3} \frac{1}{\Omega} \]
Series Series Feedback Amplifier
Shunt Series

$V_{\text{plus}} := 15V$

$V_{\text{minus}} := -15V$

$R_1 := 10k\Omega$

$R_2 := 500k\Omega$

$R_3 := 100k\Omega$

$R_4 := 100\Omega$

$R_5 := 8.7k\Omega$

$R_6 := 21.0375k\Omega$

$R_7 := 2.3375k\Omega$

$R_8 := 0.5 mA$

$R_9 := 2.5k\Omega$

$R_L := 10k\Omega$

$V_{\text{BE}} := 0.65V$

$V_T := 25.9mV$

$R_p(A, B) := \frac{A-B}{A+B}$

$V_{B1} := V_{\text{plus}}\left(\frac{R_3 - R_2}{R_3 + R_2}\right) = -10V$

$V_{E1} := V_{B1} - V_{\text{BE}} = -10.65V$

$V_{E2} := V_{C1} - V_{\text{BE}} = -5.65V$

$R_G := 0\Omega$

$I_1 := \frac{V_{E1} - V_{\text{minus}}}{R_5} = 0.5\,\text{mA}$

$I_2 := \frac{V_{E2} - V_{\text{minus}}}{R_7} = 4\,\text{mA}$

$\beta := 10^{307}$

$R_{67} := R_p(R_6, R_7)$

$R_F := R_6$

$R_{E2} := R_{67}$

$R_{C1} := R_8$

$V_{\text{ou}}$
\[ R_{23} := R_p(R_2, R_3) \quad R_1 := R_p(R_1, R_{23}) \]

\[
A := R_p(R_1, R_{F} + R_{E2}) \left[ G_{m1} \left( -G_{m2} R_C \right) - \frac{G_{m2} R_{E2}}{R_F + R_{E2}} \right] = -810.856
\]

\[
b := \frac{R_{E2}}{R_F + R_{E2}} = -0.091 \quad \frac{1}{b} = -11
\]

\[
A_1 := \frac{A}{1 + b \cdot A} = -10.853
\]
Shunt Series Feedback Amplifier