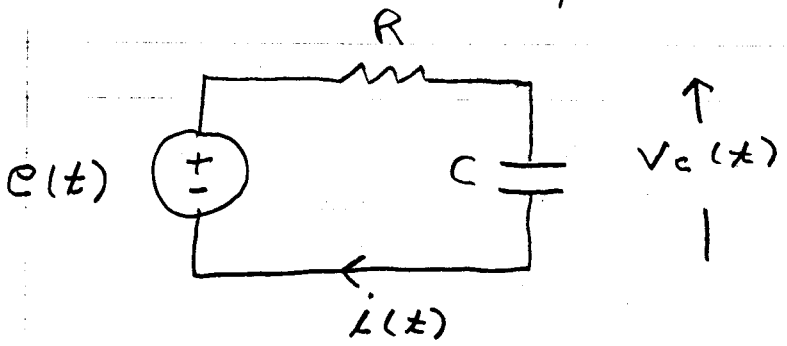
Transient Response of 1st Order RC Circuit



$$e(t) = R\,i(t) + v_c(t)$$

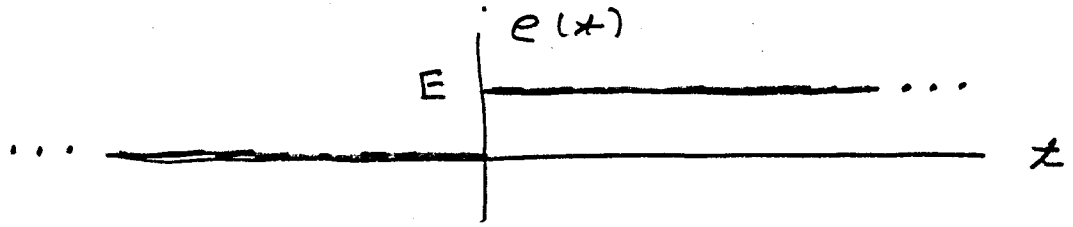
$$i(t) = C\,\frac{dv_c(t)}{dt}$$

τ = RC time constant

$$\tau\,\frac{dv_c(t)}{dt} + v_c(t) = e(t)$$

Step Function Response

$$e(t) = E\,u(t)$$



$$v_c(t) = B\,e^{-\frac{t}{\tau}} + E \qquad t \geq 0$$

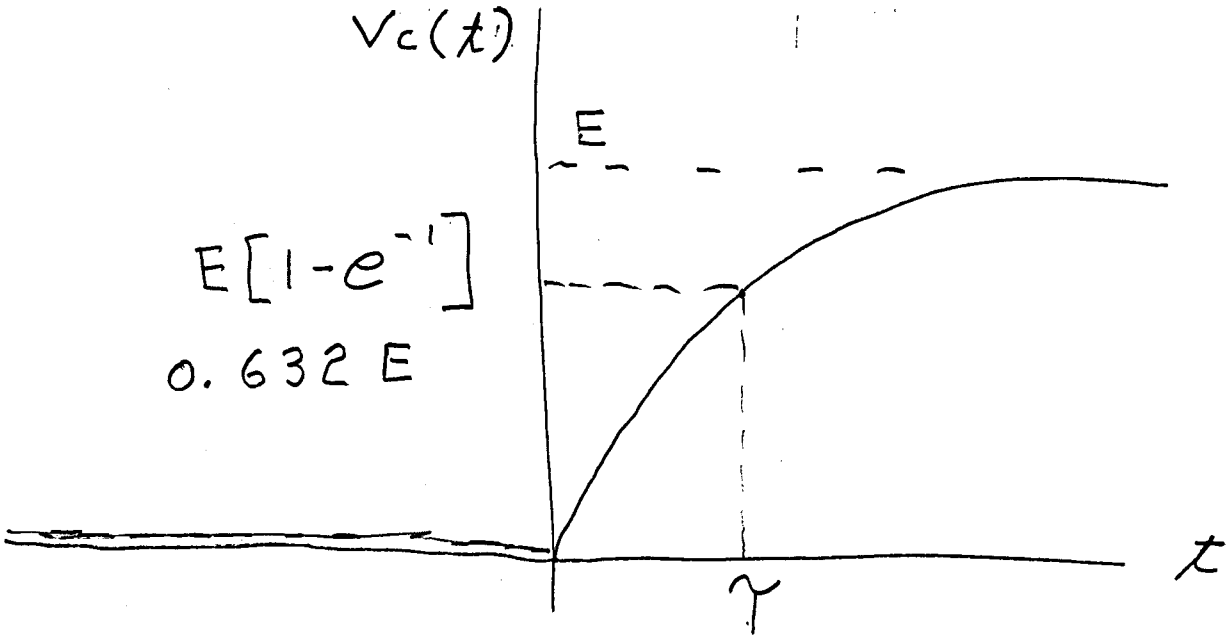
$B\,e^{-\frac{t}{\tau}}$: homogeneous solution; E: particular solution

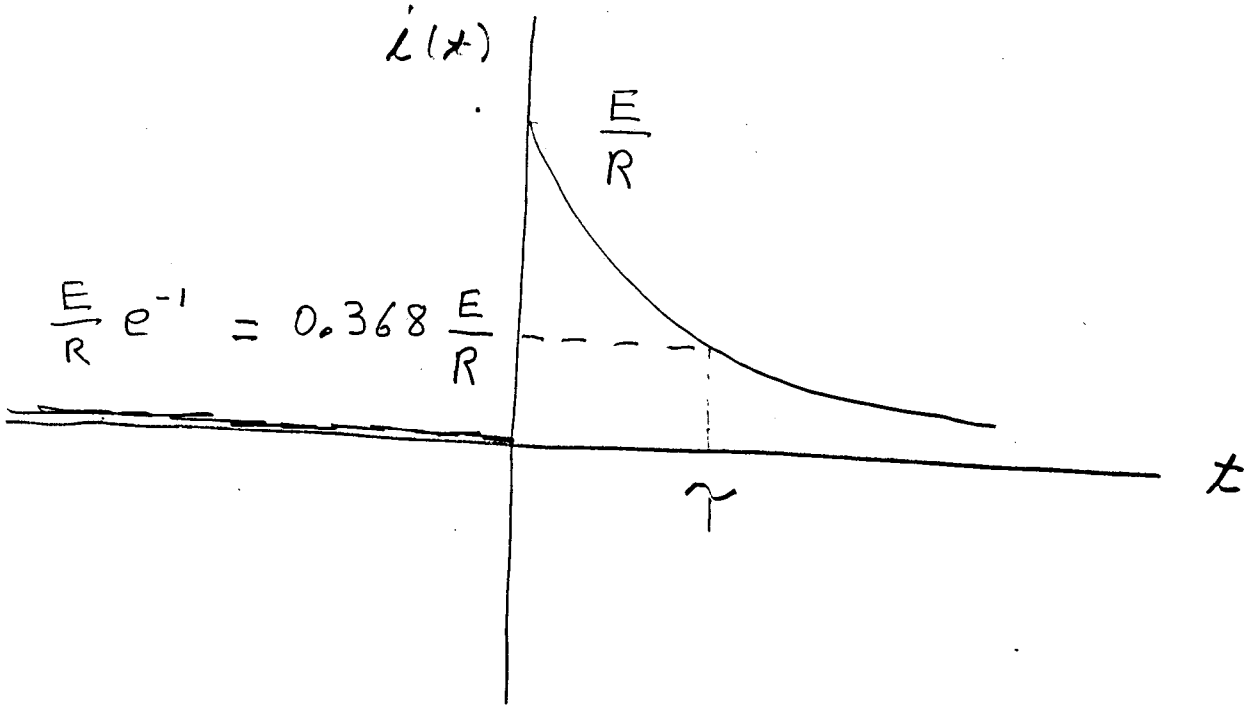
$$v_c(0) = 0 \Rightarrow v_c(t) = E\left[1 - e^{-\frac{t}{\tau}}\right] u(t)$$

$$i(t) = \frac{E}{R}\,e^{-\frac{t}{\tau}}\,u(t)$$

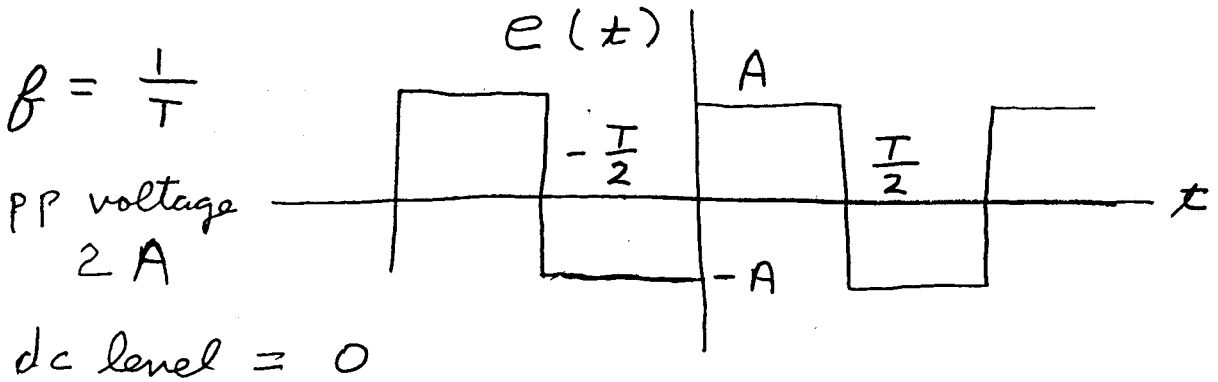
$v_c(t)$ is continuous for all t

$i(t)$ has a discontinuity at $t = 0$


Vc(t)
E
E[1-e⁻¹]
0.632 E
t
τ


i(t)
E/R
E/R e⁻¹ = 0.368 E/R
t
τ

Square Wave Response



$$\tau \frac{dv_c(t)}{dt} + v_c(t) = e(t)$$

$$e(t) = \begin{cases} \div A & -\frac{T}{2} < t < 0 \\ 0 & t = 0 \\ A & 0 < t < \frac{T}{2} \end{cases}$$

$e(t \pm n T) = e(t)$ periodic with period T

$\Rightarrow$ $v_c(t)$ must be periodic with period T

$v_c(t)$ must be a continuous function of time

$$\tau \frac{dv_c(t)}{dt} + v_c(t) = \begin{cases} -A & -\frac{T}{2} < t < 0 \\ A & 0 < t < \frac{T}{2} \end{cases}$$

$$v_c(t) = \begin{cases} B_1 e^{-\frac{t}{\tau}} - A & -\frac{T}{2} \le t \le 0 \\ B_2 e^{-\frac{t}{\tau}} + A & 0 \le t \le \frac{T}{2} \end{cases}$$

$$v_c(0) = B_1 - A = B_2 + A$$

$$v_c\left(-\frac{T}{2}\right) = B_1 e^{\frac{T}{2\tau}} - A = v_c\left(\frac{T}{2}\right)$$

$$v_c\left(\frac{T}{2}\right) = B_2 e^{-\frac{T}{2\tau}} + A$$

$$v_c(t) = \begin{cases} \left[A + v_c(0)\right] e^{-\frac{t}{\tau}} - A & -\frac{T}{2} \le t \le 0 \\ \left[-A + v_c(0)\right] e^{-\frac{t}{\tau}} + A & 0 \le t \le \frac{T}{2} \end{cases}$$

$$v_c(0) = -A \tanh \frac{T}{4\tau}$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Square Wave Response of First Order RC Circuit

$A := 10 \quad f := 10^3 \quad T := \frac{1}{f} \quad R := 10^3 \quad C := 0.22 \cdot 10^{-6} \quad \tau := R \cdot C$

$v_{co} := -A \cdot \tanh\left(\frac{T}{4 \cdot \tau}\right) \quad v_{co} = -8.132 \quad N := 2000 \quad i := 0 .. N - 1 \quad t_{start} := -T \quad t_{stop} := T$

$t_i := t_{start} + \left(t_{stop} - t_{start}\right) \cdot \frac{i}{N - 1}$ Linearly spaced points.

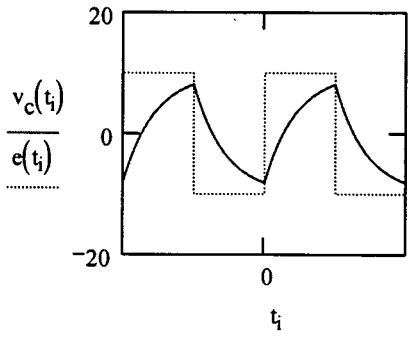
$l(t) := \left(A + v_{co}\right) \cdot e^{\frac{-t}{\tau}} - A$ The cap voltage for -T/2 to 0.

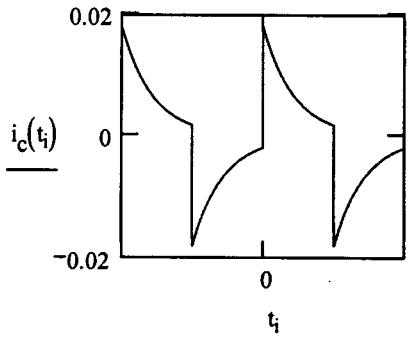
$r(t) := \left(-A + v_{co}\right) \cdot e^{\frac{-t}{\tau}} + A$ The cap voltage for 0 to T/2

$$p(t) := \begin{cases} l(t) & \text{if } \frac{-T}{2} < t < 0 \\ r(t) & \text{if } 0 \le t \le \frac{T}{2} \\ 0 & \text{otherwise} \end{cases} \qquad s(t) := \begin{cases} -1 & \text{if } \frac{-T}{2} < t < 0 \\ 1 & \text{if } 0 < t < \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$

Define one cycle of cap voltage and source

$v_c(t) := \sum_{n=-2}^{2} p(t - n \cdot T) \qquad e(t) := A \cdot \sum_{n=-2}^{2} s(t - n \cdot T) \qquad i_c(t) := \frac{e(t) - v_c(t)}{R}$





$v_c(0) = -8.132$