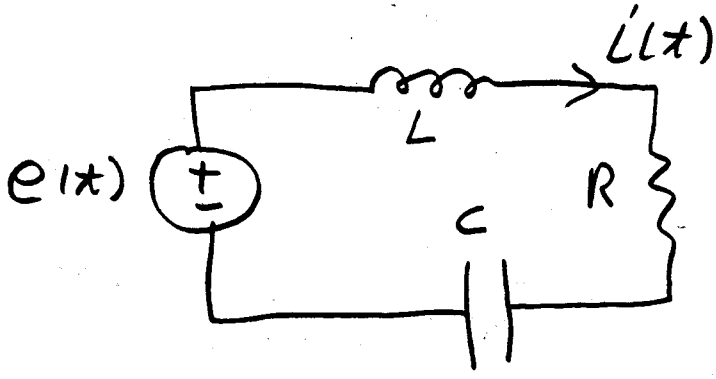
Second Order Circuits



$$i'(t) = i_c(t) = C \frac{dV_c(t)}{dt}$$

$$V_L(t) + V_R(t) + V_c(t) = e(t) \qquad \text{KVL}$$

$$L \frac{di'(t)}{dt} + R\, i'(t) + V_c(t) = e(t)$$

$$LC \frac{d^2 V_c(t)}{dt^2} + RC \frac{dV_c(t)}{dt} + V_c(t) = e(t)$$

$$\frac{d^2 V_c(t)}{dt^2} + \frac{R}{L} \frac{dV_c(t)}{dt} + \frac{1}{LC} V_c(t) = \frac{e(t)}{LC}$$

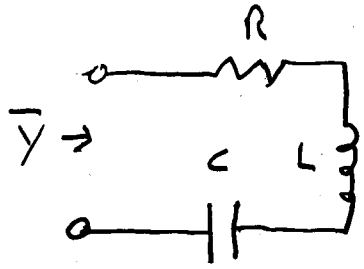
$$\frac{d^2 V_c(t)}{dt^2} + 2\rho\omega_0 \frac{dV_c(t)}{dt} + \omega_0^2 V_c(t) = \frac{e(t)}{LC}$$

$$\rho = \frac{R}{2}\sqrt{\frac{C}{L}} \equiv \text{damping factor}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \equiv \text{natural or resonant frequency}$$

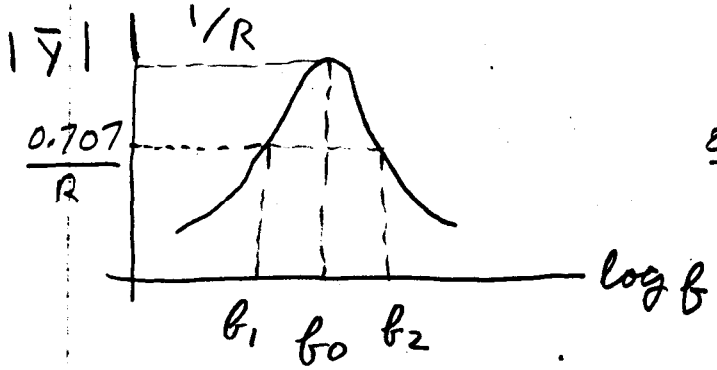
Sinusoidal Excitation ∿

Series RLC



Resonance

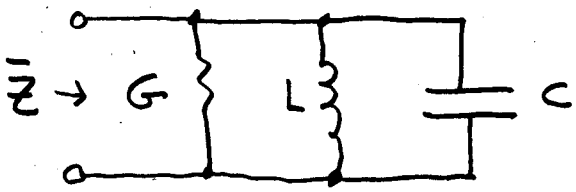
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$



$$\Delta f = BW = f_2 - f_1 \equiv \text{Half-power bandwidth}$$

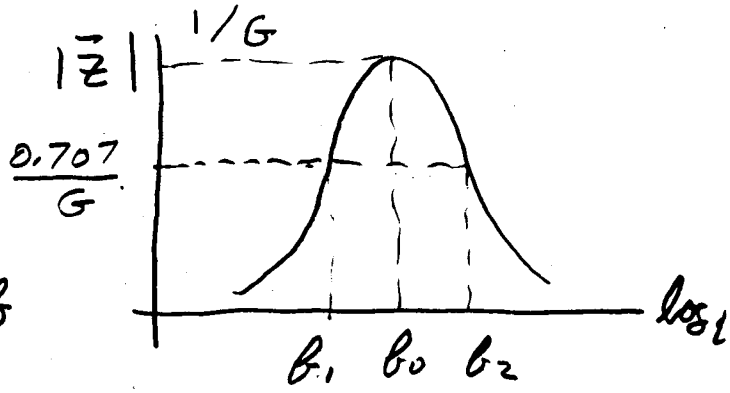
$$Q = \frac{f_0}{\Delta f} = \frac{\omega_0 L}{R}$$

Parallel GCL



Resonance

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$



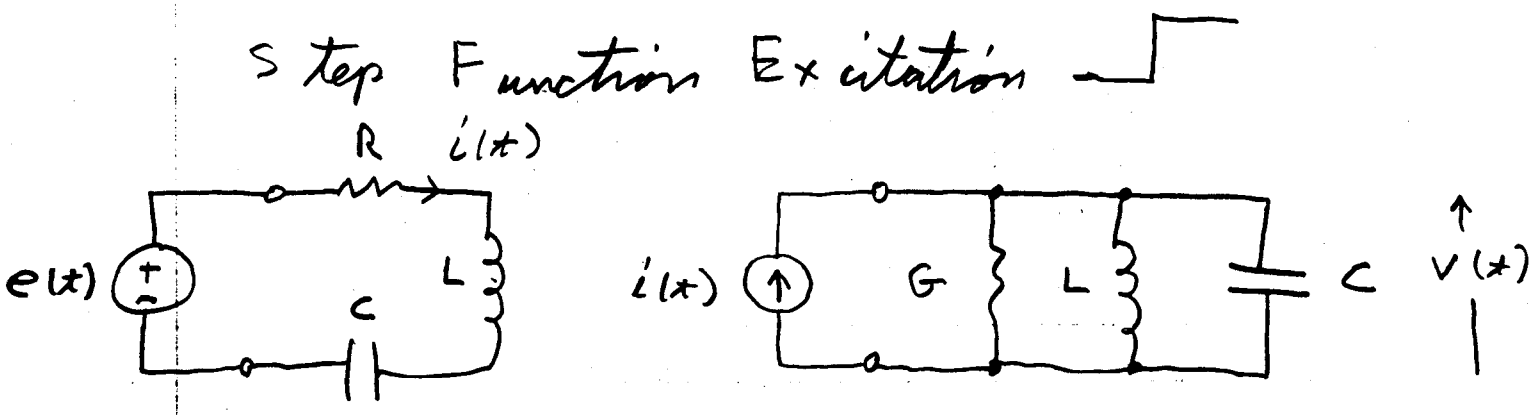
$$\equiv BW = f_2 - f_1 = \Delta f$$

$$Q = \frac{f_0}{\Delta f} = \frac{\omega_0 C}{G}$$

Duality

Series RLC		Parallel GCL
R	⟷	G
L	⟷	C
C	⟷	L
$\bar{Y}$	⟷	$\bar{Z}$
e	⟷	i
i	⟷	e

Step Function Excitation

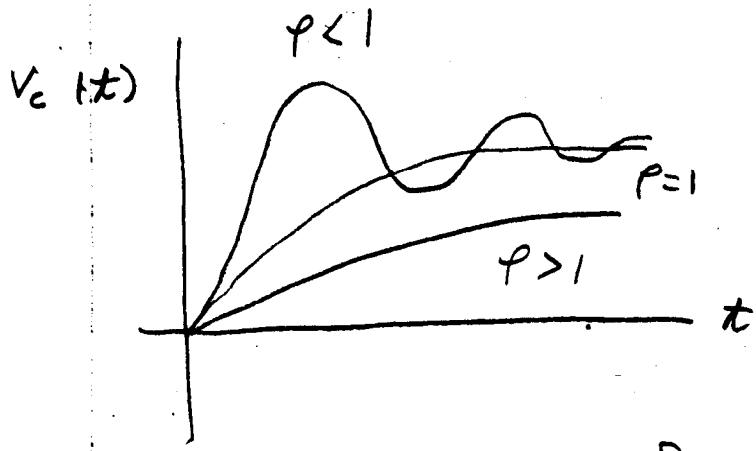


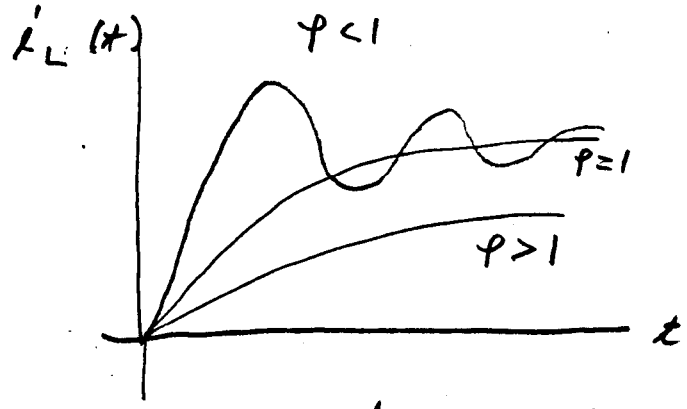
Series

$$\rho = \frac{R}{2}\sqrt{\frac{C}{L}}$$

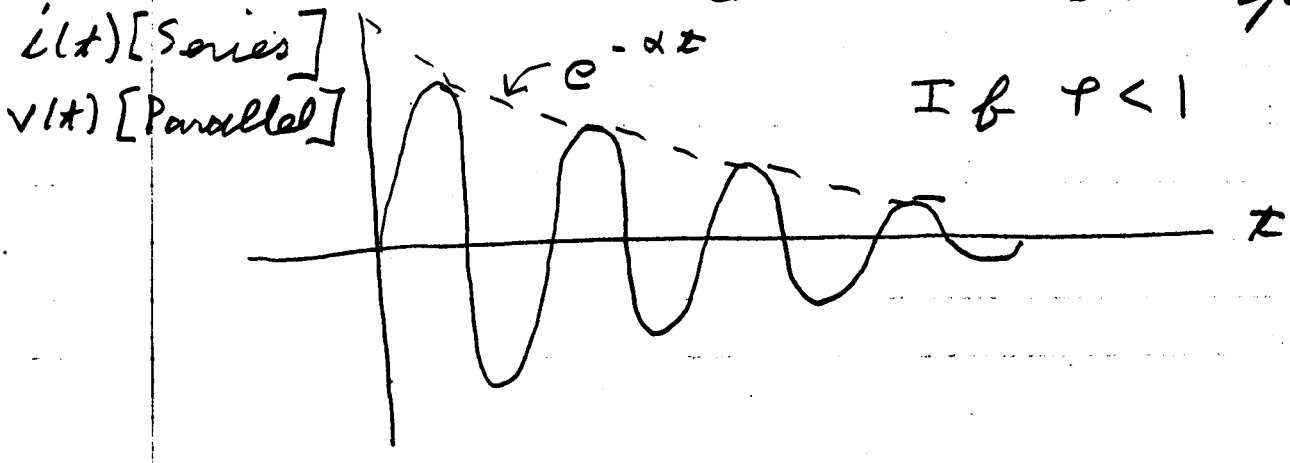
Parallel

$$\rho = \frac{G}{2}\sqrt{\frac{L}{C}}$$





$$\rho = \begin{cases} <1 & \text{underdamped} \\ =1 & \text{critically damped} \\ >1 & \text{overdamped} \end{cases}$$



$\alpha \equiv$ attenuation factor of the envelope

$$\alpha = \frac{R}{2L}\ [\text{Series}] \qquad \alpha = \frac{G}{2C}\ [\text{Parallel}]$$

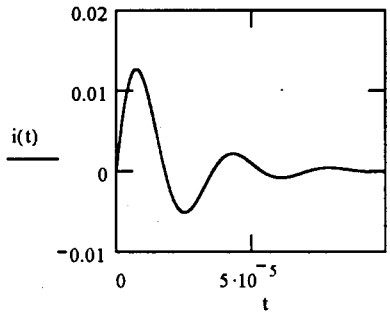
$R := 300 \qquad L := 3\cdot10^{-3} \qquad C := 10^{-8} \qquad E := 10$

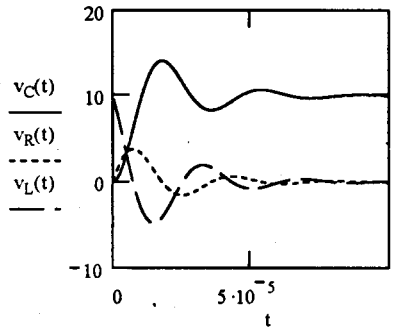
$\zeta := \frac{R}{2}\cdot\sqrt{\frac{C}{L}} \qquad \zeta = 0.274$ Underdamped $\qquad u(t) := \Phi(t)$ $\Phi()$ in Matlab step function

$\alpha := \frac{R}{2\cdot L} \qquad \omega_o := \frac{1}{\sqrt{L\cdot C}} \qquad \omega_d := \omega_o\cdot\sqrt{1-\zeta^2} \qquad i(t) := \frac{E}{R}\frac{2\zeta}{\sqrt{1-\zeta^2}}e^{-\alpha\cdot t}\cdot\sin(\omega_d\cdot t)\cdot u(t)$

$v_R(t) := R\cdot i(t) \qquad v_L(t) := L\cdot\frac{d}{dt}i(t) \qquad e(t) := E\cdot u(t)$

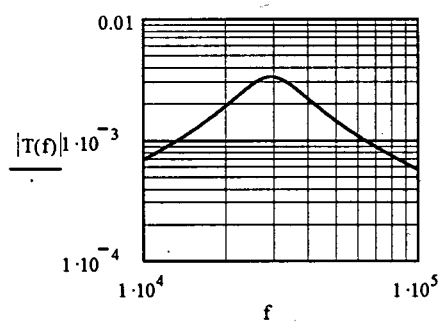
$v_C(t) := e(t) - v_R(t) - v_L(t)$

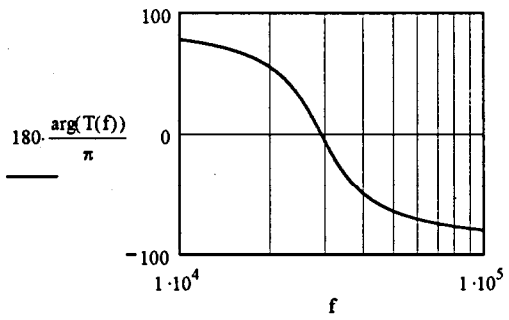


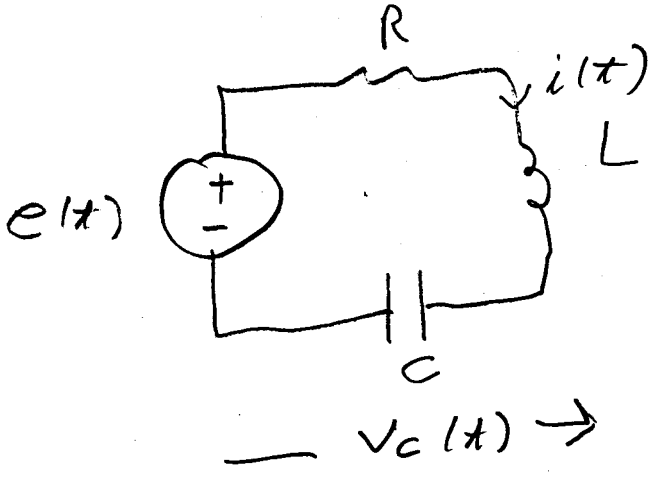


$j := \sqrt{-1} \qquad f_o := \frac{\alpha}{2\cdot\pi} \qquad Q := \frac{\omega_o\cdot L}{R}$

$T(f) := \frac{1}{R}\cdot\frac{1}{1 + j\cdot Q\cdot\left(\frac{f}{f_o} - \frac{f_o}{f}\right)}$ Admittance T(f)=I/E







$v_L(t)$

Second Order Circuit Series RLC Step Function Response

Direct Solution of Differential Equation

$E := 100 \qquad L := 0.001 \qquad C := 10^{-6} \qquad R := 15$

$\zeta := \frac{R}{2}\cdot\sqrt{\frac{C}{L}} \qquad f_n := \frac{1}{2\cdot\pi\cdot\sqrt{L\cdot C}} \qquad \omega_n := 2\cdot\pi\cdot f_n$

$\zeta = 0.237$

$f_n = 5.033 \times 10^{3} \qquad \omega_n = 3.162 \times 10^{4}$

Given

$$\frac{d^2}{dt^2}v(t) = -2\cdot\zeta\cdot\omega_n\cdot\left(\frac{d}{dt}v(t)\right) - \omega_n{}^2 v(t) + \omega_n{}^2\cdot E \qquad v(0) = 0 \qquad v'(0) = 0$$

$v := \text{Odesolve}(t, 0.001)$

$i(t) := C\cdot\left(\frac{d}{dt}v(t)\right) \qquad v_L(t) := L\cdot\left(\frac{d}{dt}i(t)\right)$

$v_R(t) := R\cdot i(t)$

