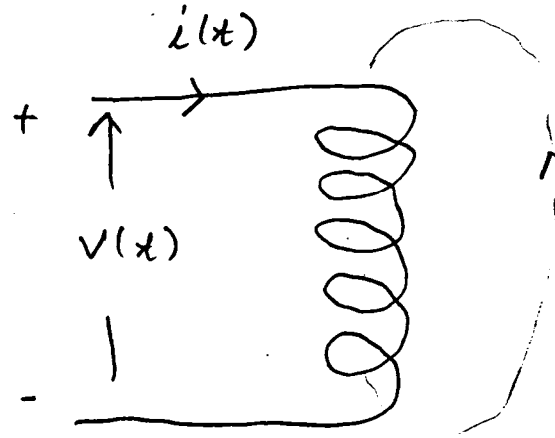
Inductor



$\Phi(t) \equiv$ magnetic flux

N turns of wire

$\lambda(t) = N \Phi(t)$

flux linkages

$$v(t) = \frac{d\lambda(t)}{dt}$$ Faraday's Law

$$v(t) = N \frac{d\Phi(t)}{dt}$$

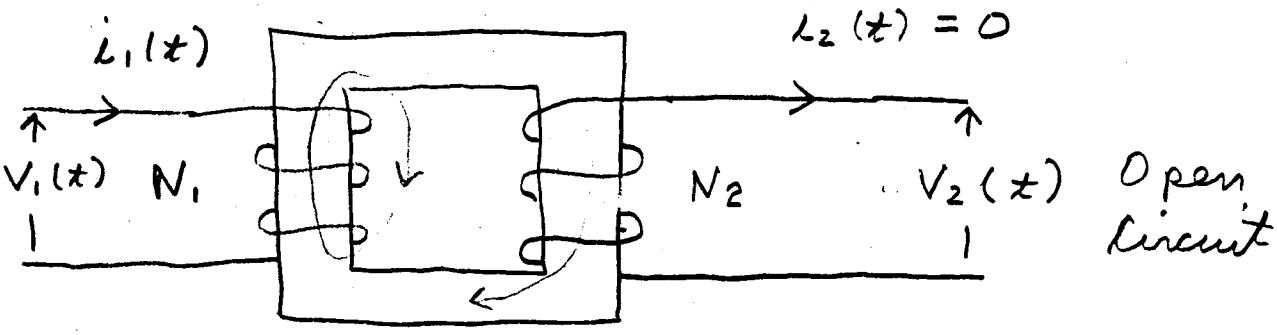
$$\Phi(t) = \mathcal{P} N i(t)$$ Ampere's Law

$\mathcal{P} \equiv$ permeance of the space occupied by flux

$$v(t) = \mathcal{P} N^2 \frac{di(t)}{dt}$$

$L \equiv$ inductance $= \mathcal{P} N^2$

Primary Transformer Secondary



$$V_2(t) = \frac{d\,\lambda_2(t)}{dt}$$

$$\lambda_2(t) = N_2\,\Phi_2(t)$$

$$\Phi_2(t) = \mathcal{P}_{21}\,N_1\,i_1(t)$$

$$V_2(t) = \mathcal{P}_{21}\,N_1\,N_2\,\frac{d\,i_1(t)}{dt}$$

$$V_2(t) = m\,\frac{d\,i_1(t)}{dt}$$

$m \equiv$ mutual inductance $= k\sqrt{L_1 L_2}$

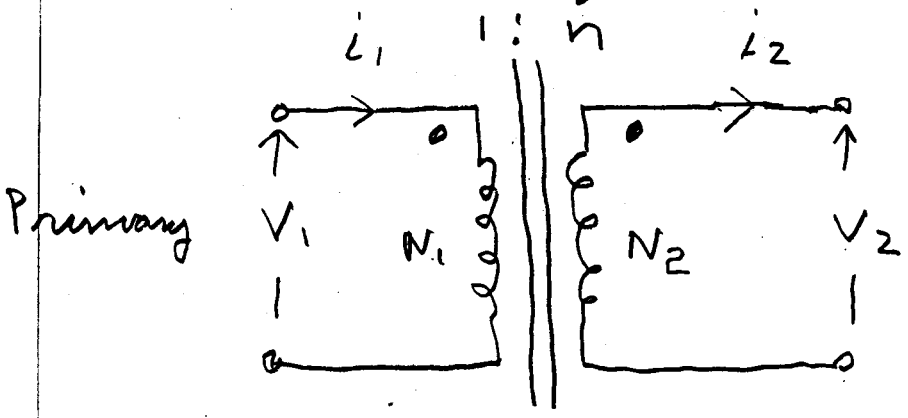
$0 \le k \le 1$ coefficient of coupling

User of transformers

1. changing voltage and current leve
2. isolation
3. impedance matching

Transformers

Ideal Transformer



$$n = \frac{N_2}{N_1} \equiv \text{turns ratio}$$

$$V_2 = n V_1 \qquad i_2 = i_1 / n$$

$$n = \frac{N_2}{N_1} = \begin{cases} > 1 & \text{step up} \\ = 1 & \text{isolation} \\ < & \text{step down} \end{cases}$$

Primary & Secondary are isolated

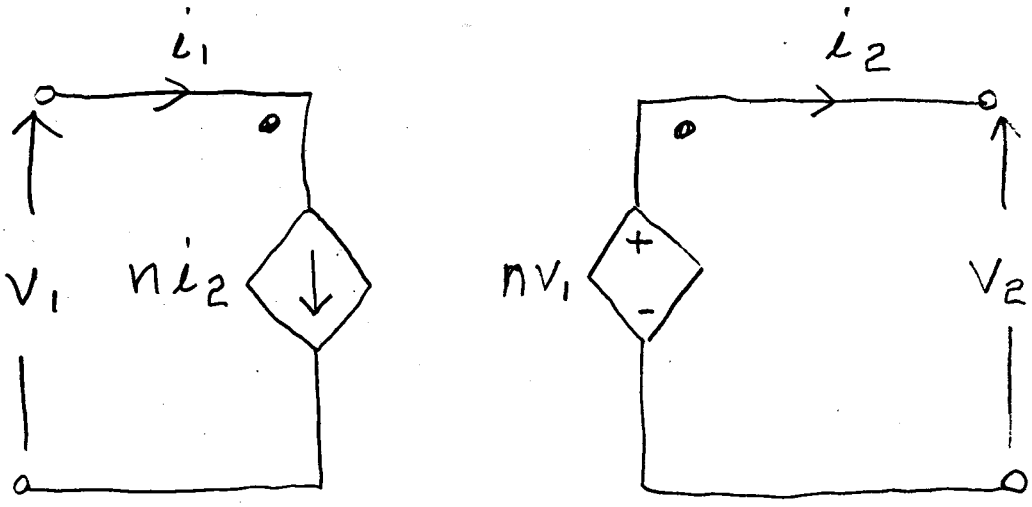
- Dot convention

The voltage at the dotted ends of the primary & secondary have the same polarity.

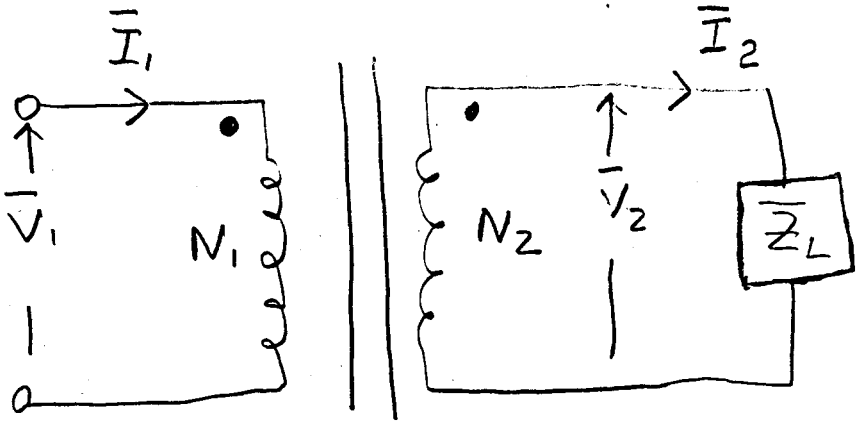
Ideal Transformer is lossless

$$P_1 = V_1 i_1 = V_2 i_2$$

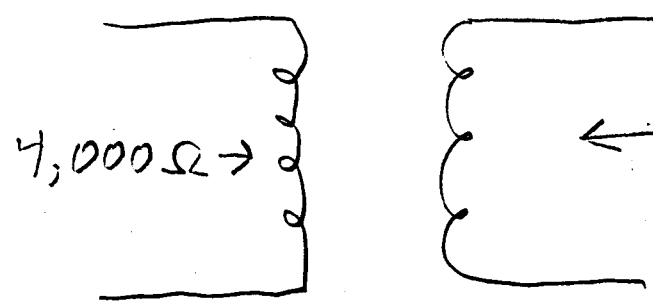
Controlled Source Model



Impedance Matching



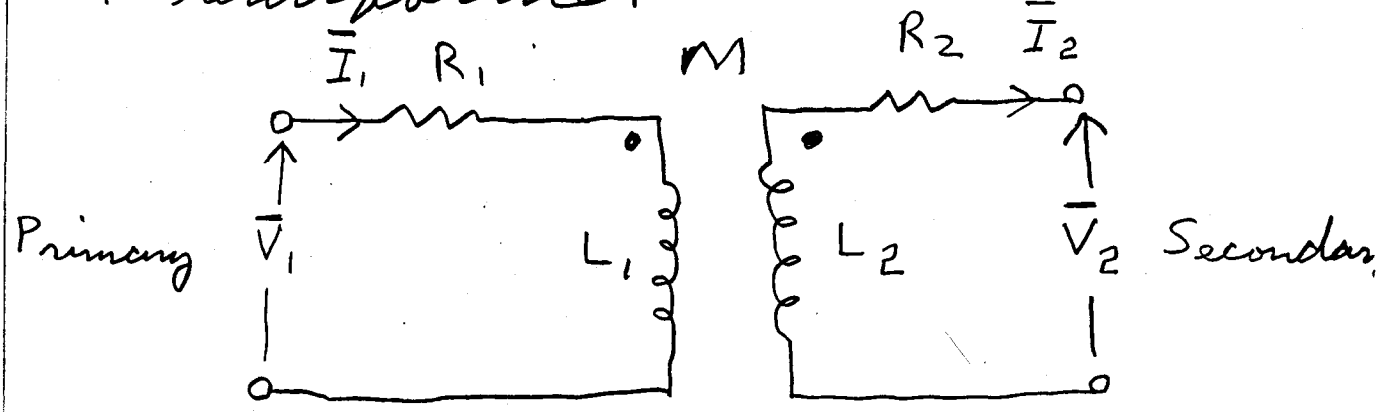
$$\bar{Z}_i = \frac{\bar{V}_1}{\bar{I}_1} = \frac{\bar{V}_2}{n} \quad \frac{1}{n\bar{I}_2} = \frac{1}{n^2}\frac{\bar{V}_2}{\bar{I}_2} = \frac{\bar{Z}_L}{n^2}$$



$$n = \sqrt{\frac{600}{4000}}$$

$$= 0.387$$

Linear Model for Non Ideal Transformer



$R_1 \equiv$ resistance of primary
$L_1 \equiv$ inductance of primary
$R_2 \equiv$ resistance of secondary
$L_2 =$ inductance of secondary

$$k \equiv \frac{M}{\sqrt{L_1 L_2}} \equiv \text{coefficient of coupling}$$

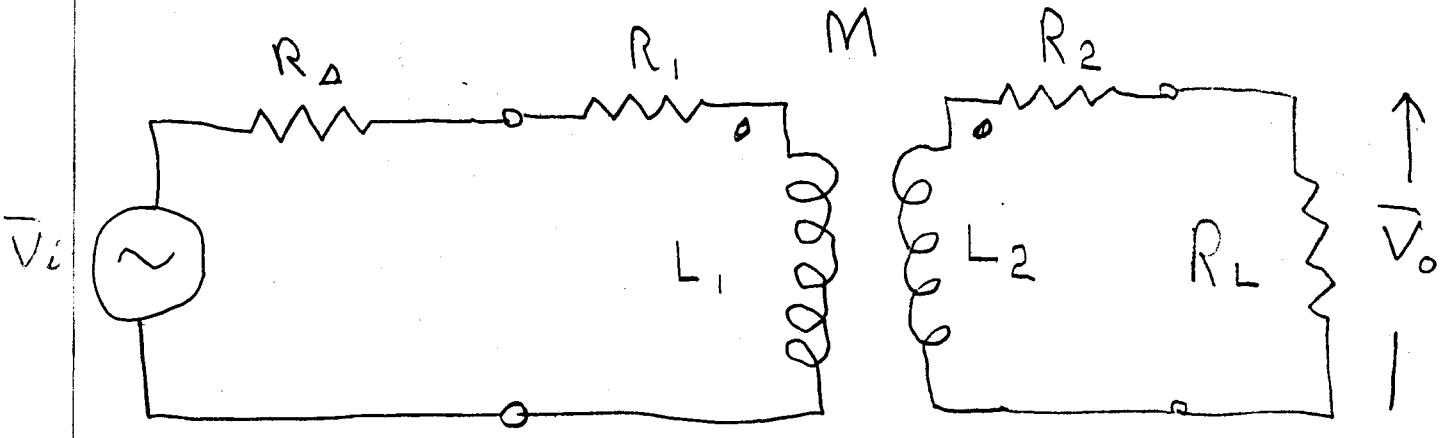
$$L_1 \propto N_1^2 \qquad L_2 \propto N_2^2$$

$$n = \frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}}$$

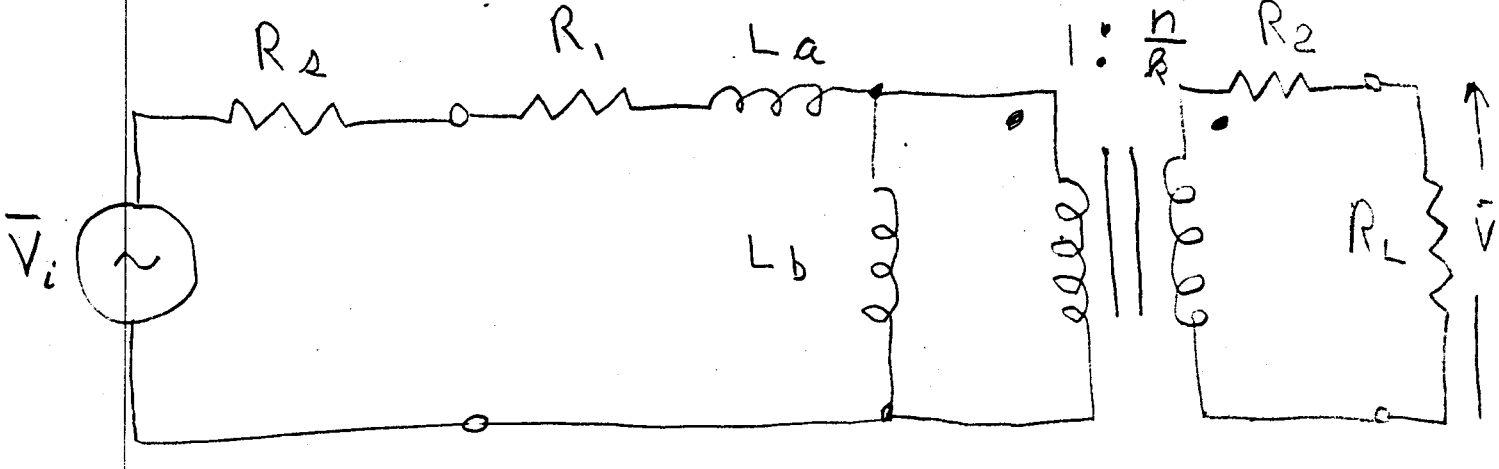
$$\bar{V}_1 = (R_1 + j\omega L_1)\bar{I}_1 - j\omega M \bar{I}_2$$

$$\bar{V}_2 = j\omega M \bar{I}_1 - (R_2 + j\omega L_2)\bar{I}_2$$

Becomes ideal transformer if
$k = 1$, $R_1 = R_2 = 0$
$L_1 = L_2 = \infty$

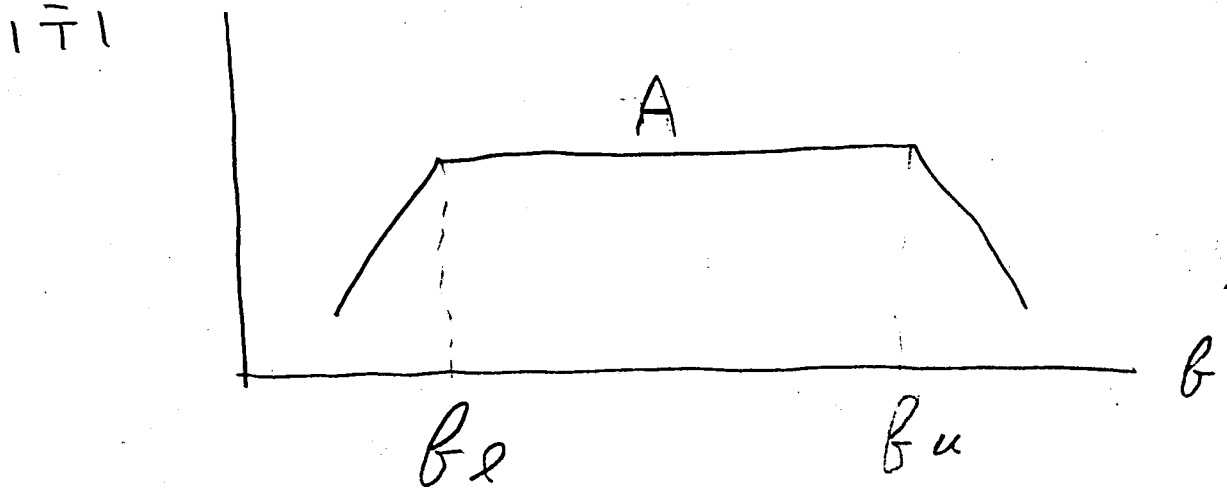


Want $\bar{T}(s) = \dfrac{\bar{V}_o(s)}{\bar{V}_i(s)}$

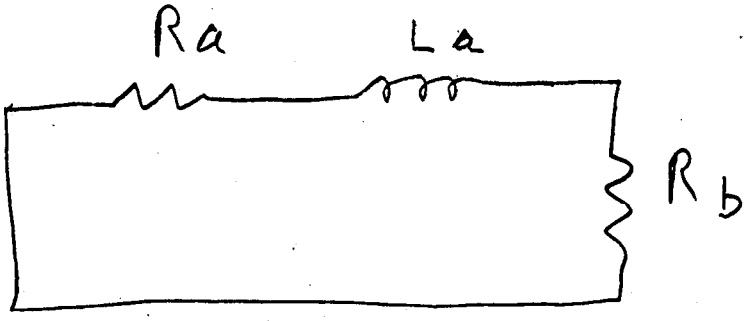


$L_a = (1-k^2) L_1 \qquad L_b = k^2 L_1$

If $k \approx 1$, this has a broadband bandpass characteristic



To get the upper pole



$$\tau_u = \frac{L_a}{R_a + R_b}, \quad f_u = \frac{1}{2\pi \tau_u}$$

$$\overline{T} \approx \frac{A \, s \tau_e}{(1 + s\tau_e)(1 + s\tau_u)}$$

$$A = \frac{R}{n} \quad \frac{R_L}{R_a + R_b}$$

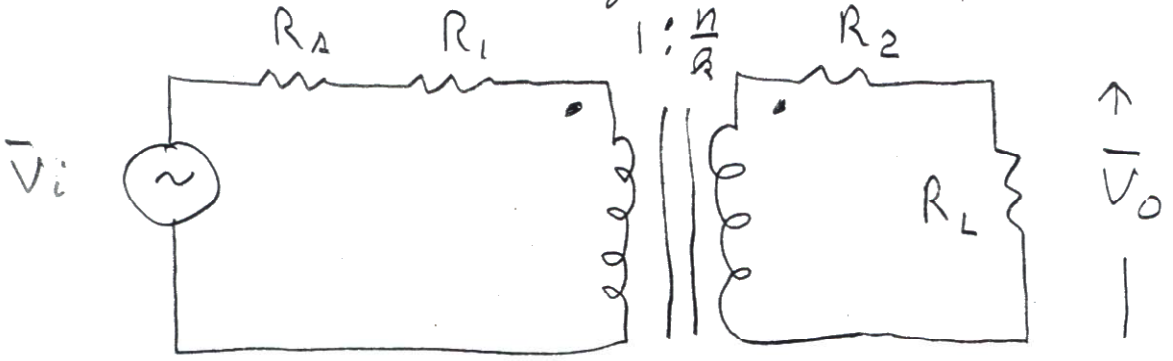
$$\tau_e = \frac{L_b}{R_a \| R_b}, \quad \tau_u = \frac{L_a}{R_a + R_b}$$

$$L_b = k^2 L_1 \qquad L_a = (1 - k^2) L_1$$

$$R_a = R_a + R_1, \quad R_b = (R_2 + R_L)\frac{k^2}{n^2}$$

$$n = \frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}}$$

For the mid frequencies $s L_a$ is small & $s L_b$ is large

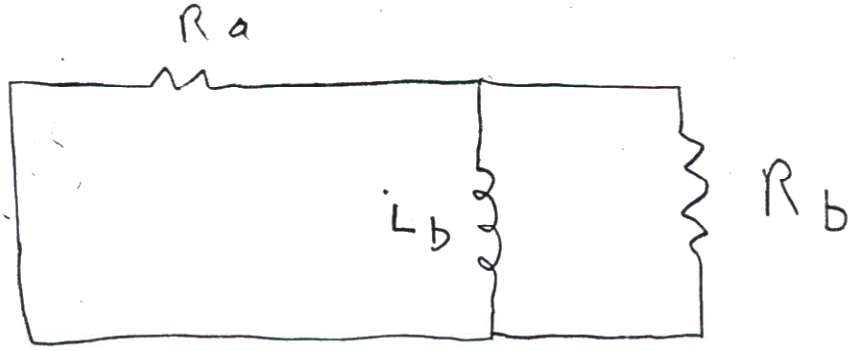


$$\bar{T} = \frac{\bar{V}_o}{\bar{V}_i} = \frac{R_b}{R_a + R_b} \cdot \frac{n}{k} \cdot \frac{R_L}{R_2 + R_L}$$

$$R_a = R_2 + R_1 \quad , \quad R_b = (R_2 + R_L)\frac{k^2}{n^2}$$

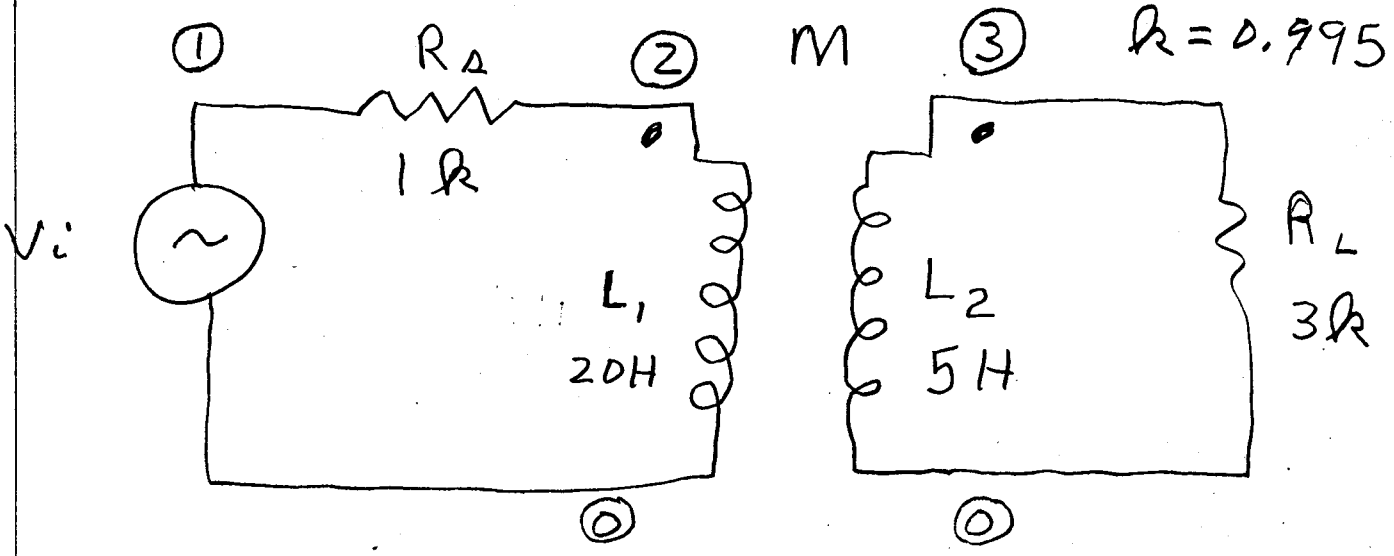
$$\bar{T} = A = \frac{k}{n} \cdot \frac{R_L}{R_a + R_1 + \frac{k^2}{n^2}(R_2 + R_L)}$$

To get the lower pole



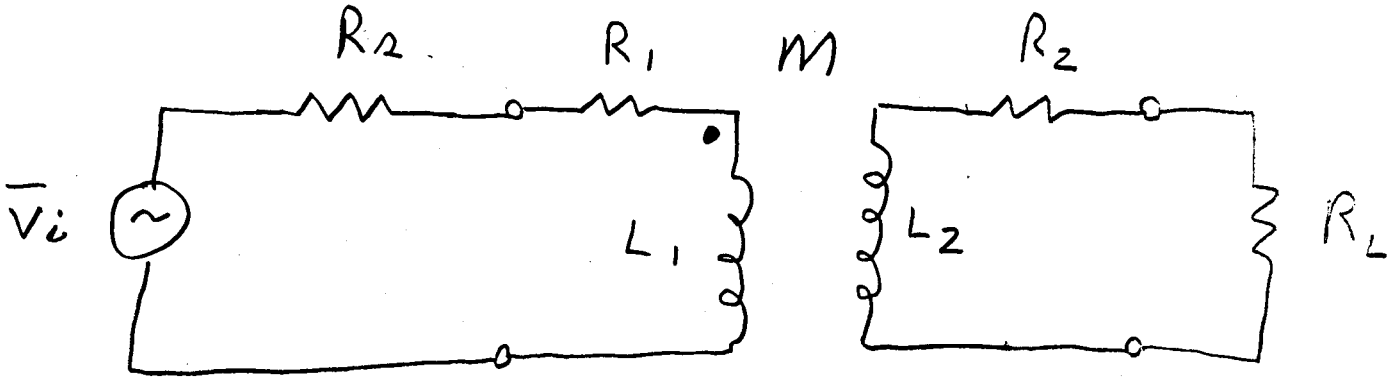
$$\tau_l = \frac{L_b}{R_a \,||\, R_b} \quad ; \quad f_l = \frac{1}{2\pi\tau_l}$$

SPICE



George P. Burdell

```
VI    1    0    AC  1
RS    1    2    1K
L1    2    0    20
L2    3    0    5
1T    L1   L2   0.995
RL    3    0    3K
.AC DEC 100 1 100K
.PROBE
.END
```



$$\bar{T} = \frac{\frac{k}{n}\,\frac{R_L}{L_a}\,s}{s^2 + s\left(\frac{R_a}{L_a} + \frac{R_b}{L_b} + \frac{R_b}{L_a}\right) + \frac{R_a R}{L_a L_t}}$$

Exact Transfer Function where

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$R_a = R_2 + R_1$$

$$R_b = \frac{k^2}{n^2}(R_2 + R_L)$$

$$L_a = (1 - k^2) L_1$$

$$L_b = k^2 L_1$$

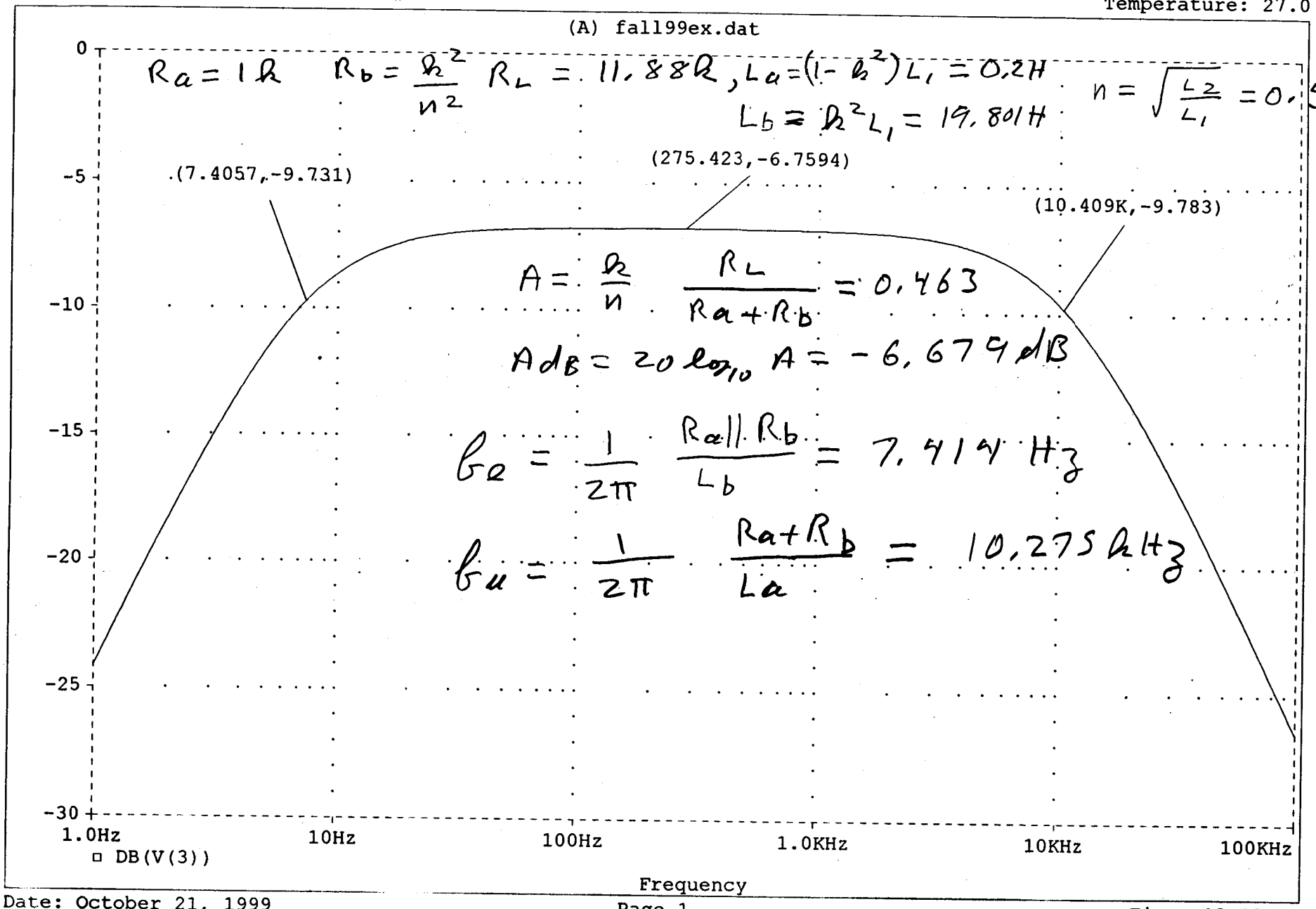
$$\bar{T}(275.423) = 0.459 \angle 0.006°$$

$$|T|_{dB} = 20 \log_{10} 0.459 = -6.759\ dB$$

George P. Burdell

Date/Time run: 10/21/99 13:12:02

Temperature: 27.0



$R_a = 1k \quad R_b = \frac{k^2}{n^2} \quad R_L = 11.88k, \; L_a = (1-k^2)L_1 = 0.2H$

$L_b = k^2 L_1 = 19.801H$

$n = \sqrt{\frac{L_2}{L_1}} = 0.5$

$A = \frac{k}{n} \cdot \frac{R_L}{R_a + R_b} = 0.463$

$A_{dB} = 20 \log_{10} A = -6.674 dB$

$f_\ell = \frac{1}{2\pi} \cdot \frac{R_a \| R_b}{L_b} = 7.414 Hz$

$f_u = \frac{1}{2\pi} \cdot \frac{R_a + R_b}{L_a} = 10.275 kHz$

Date: October 21, 1999

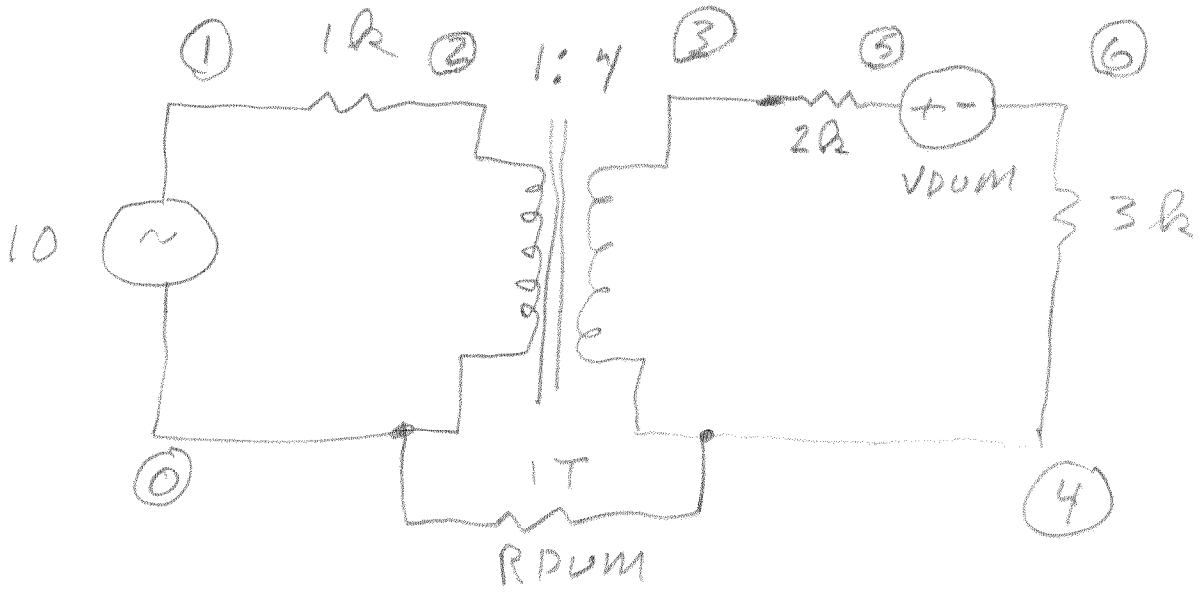
Page 1

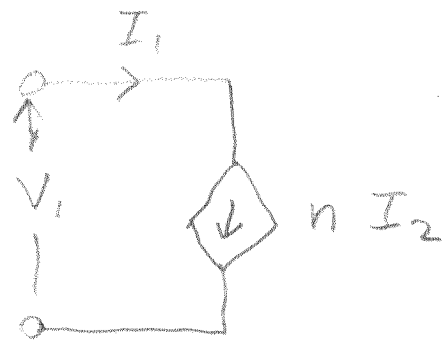
Time: 13:13:56

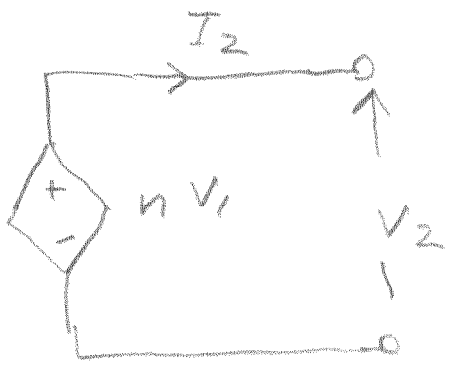
XF-10

```
Ideal Transformer Example
VI 1 0 AC 10
R1 1 2 1K
F 2 0 VDUM 4
*F IS A CURRENT CONTROLLED CURRENT SOURCE WITH A GAIN OF 4
*VDUM IS A DUMMY VOLTAGE SOURCE USED TO SENSE THE CONTROLLING CURRENT
E 3 4 2 0 4
*E IS A VOLTAGE CONTROLLED VOLTAGE SOURCE
RDUM 4 0 1T
*RDUM IS A DUMMY RESISTOR SO THAT THE NODES IN THE SECONDARY DON'T *FLOAT
R2 3 5 2K
VDUM 5 6 DC 0
RL 6 4 3K
.AC DEC 30 100 100K
.PROBE
.END
```

$n = 4$







$I_2 = \frac{I_1}{n}$

$V_2 = n V_1$

$n \equiv$ turns ratio